Numerical Study of a Jeffcott Rotor Model with a Snubber Ring

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Modern Practice in Stress and Vibration Analysis (MPSVA 2012)

UNIVERSITY OF ABERDEEN

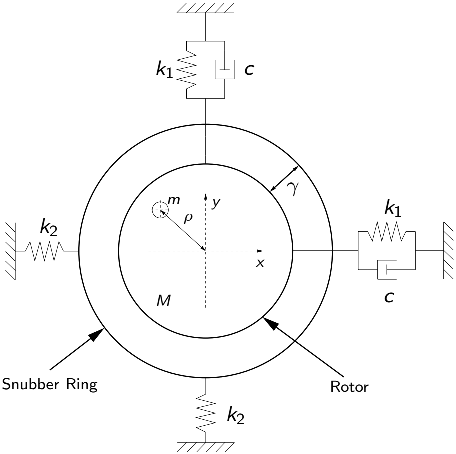
University of Glasgow

Overview

- Description of the rotor system.
- Equations of motion.
- Bifurcation analysis with TC-HAT.
- Analytical grazing curve.
- Future work.

Description of the rotor system

Physical model



Description of the rotor system
Physical model

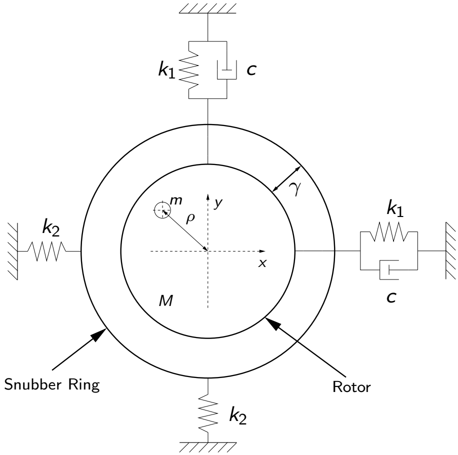
Assumptions:

- Massless snubber ring.
- No dry friction between rotor and snubber ring.
- Snubber ring elastically supported (no viscous damping).
- No gyroscopic forces.
- Gravity loads are neglected.

Imbalance caused by:

- Thermal deformation.
- Blade/tooth breakage.

$\Rightarrow$ Serious malfunctions.



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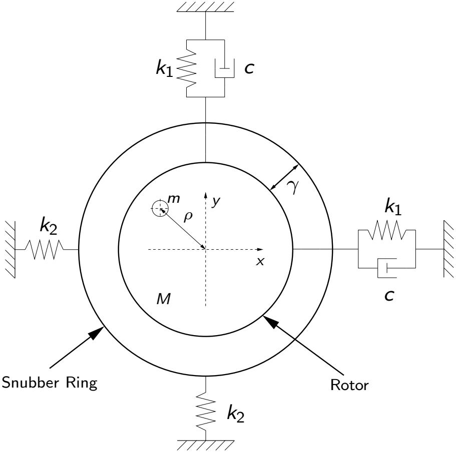

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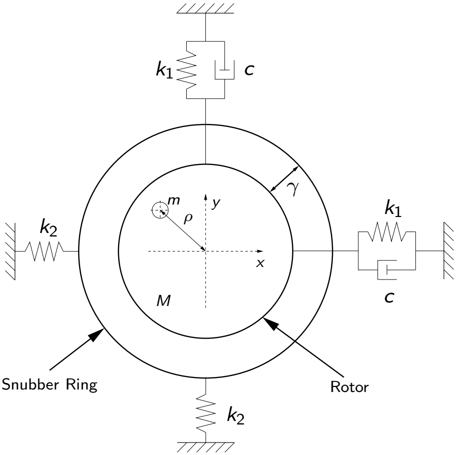
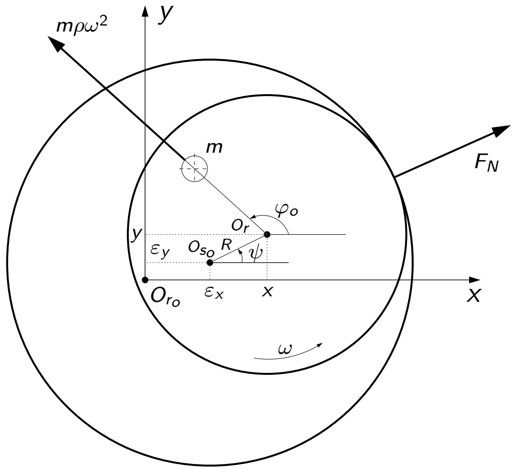


Description of the rotor system

Geometrical representation



Description of the rotor system

Geometrical representation

O_{r_0}: Rotor static position.

O_{s_0}: Snubber ring resting position.

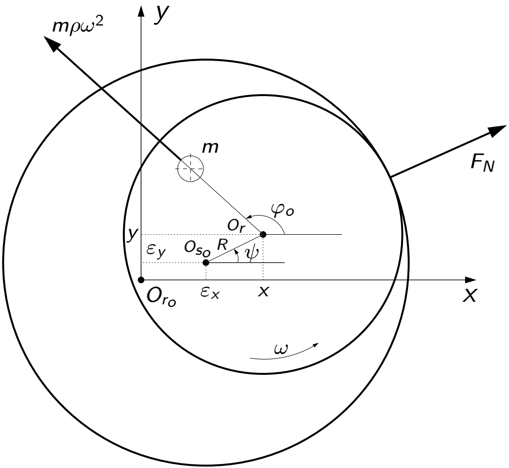
(x, y): Coordinates of the center of the rotor O_r.

$(\varepsilon_x, \varepsilon_y)$: Eccentricity of the rotor.

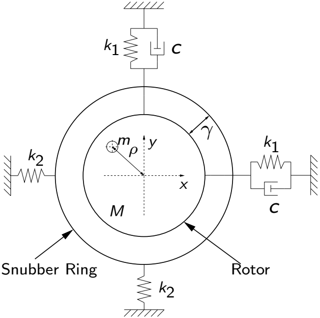
F_N: Normal force produced by contact.

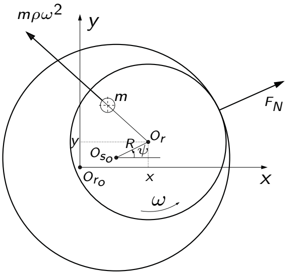
ω: Frequency of rotation.

$m\rho\omega^2$: Centrifugal force.



Equations of motion





No contact ($R < \gamma$):

$$Mx'' + cx' + k_1 x = m\rho\omega^2 \cos(\omega t + \varphi_0),$$
$$My'' + cy' + k_1 y = m\rho\omega^2 \sin(\omega t + \varphi_0).$$

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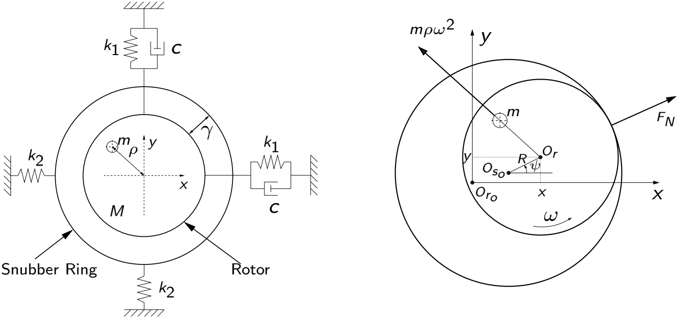

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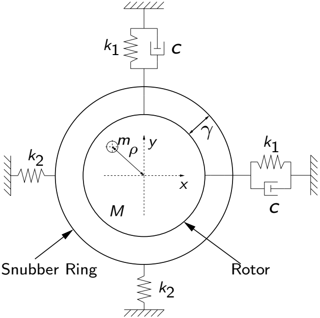

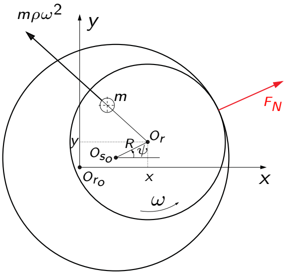

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Contact ($R \geq \gamma$):

$$Mx'' + cx' + k_1 x + k_2(R - \gamma)\cos(\psi) = m\rho\omega^2 \cos(\omega t + \varphi_0),$$

$$My'' + cy' + k_1 y + k_2(R - \gamma)\sin(\psi) = m\rho\omega^2 \sin(\omega t + \varphi_0).$$

Equations of motion
Nondimensionalization

Dimensionless variables and parameters:

$$\omega_n = \sqrt{\frac{k_1}{M}}$$
$$\eta = \frac{\omega}{\omega_n}$$
$$t \leftarrow \omega_n t$$
$$x \leftarrow \frac{x}{\gamma}$$
$$y \leftarrow \frac{y}{\gamma}$$
$$z = x'$$
$$w = y'$$
$$s = \eta t \in [0, 2\pi)$$
$$v = (x, y, z, w, s)$$

$$\varepsilon_x \leftarrow \frac{\varepsilon_x}{\gamma}$$
$$\varepsilon_y \leftarrow \frac{\varepsilon_y}{\gamma}$$
$$\eta_m = \frac{m}{M}$$
$$\nu = \frac{c}{2\sqrt{k_1 M}}$$
$$K = \frac{k_2}{k_1}$$
$$\rho \leftarrow \frac{\rho}{\gamma}$$
$$\alpha = (\eta_m, \rho, \nu, K, \varphi_0, \varepsilon_x, \varepsilon_y)$$

Equations of motion
First order ODE

$$v' = \begin{cases} f_{\text{contact}}(v, \alpha, \eta), & R \geq \gamma, \\ f_{\text{no contact}}(v, \alpha, \eta), & R < \gamma, \end{cases}$$

where

$$f_{\text{no contact}}(v, \alpha, \eta) = \begin{pmatrix} z \\ w \\ \eta_m \rho \eta^2 \cos(s + \varphi_0) - 2\nu z - x \\ \eta_m \rho \eta^2 \sin(s + \varphi_0) - 2\nu w - y \\ \eta \end{pmatrix},$$

$$f_{\text{contact}}(v, \alpha, \eta) = \begin{pmatrix} z \\ w \\ \eta_m \rho \eta^2 \cos(s + \varphi_0) - 2\nu z - x - K(x - \varepsilon_x)\left(1 - \frac{\gamma}{R}\right) \\ \eta_m \rho \eta^2 \sin(s + \varphi_0) - 2\nu w - y - K(y - \varepsilon_y)\left(1 - \frac{\gamma}{R}\right) \\ \eta \end{pmatrix}.$$

Equations of motion

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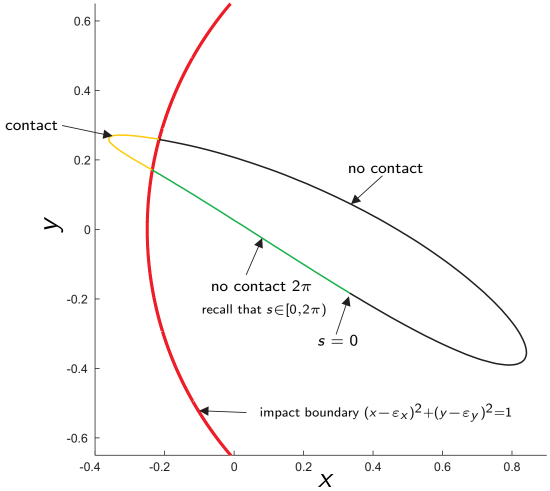
Bifurcation analysis with TC-HAT

Auto 97 driver for the continuation of periodic orbits of non-smooth systems (Thota & Dankowicz, SIAM J. Appl. Dyn. Sys., 2008)

Assume $\varepsilon_y = 0$, $\varepsilon_x \neq 0$.



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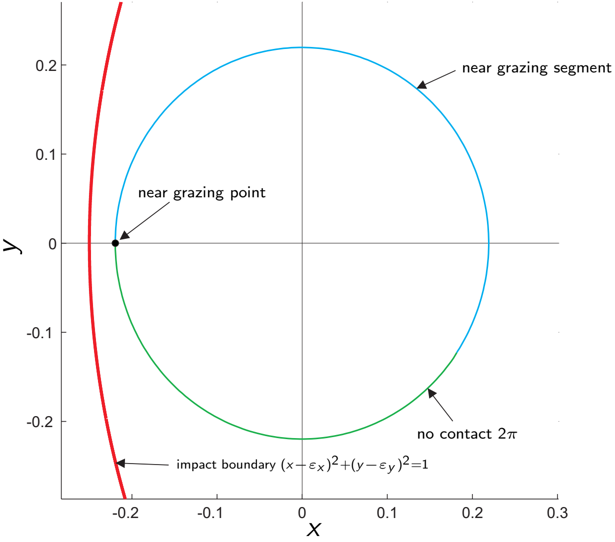


Bifurcation analysis with TC-HAT

Segments, event and jump functions

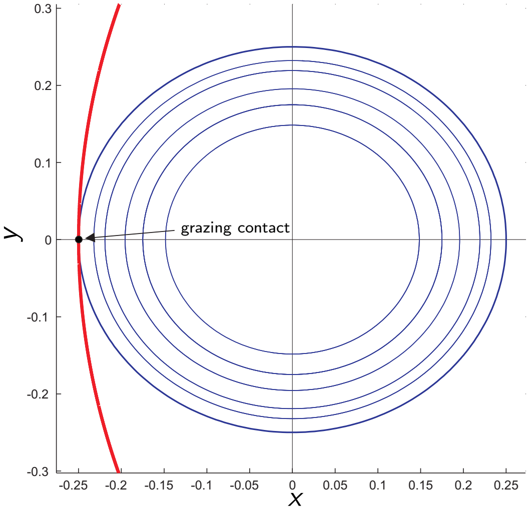
Index	Segment	Vector Field	Event	Jump
I_1	no contact	$f_{\text{no contact}}$	h_{contact}	g_{id}
I_2	contact	f_{contact}	h_{contact}	g_{id}
I_3	no contact 2π	$f_{\text{no contact}}$	$h_{2\pi}$	$g_{2\pi}$
I_4	contact 2π	f_{contact}	$h_{2\pi}$	$g_{2\pi}$
I_5	near grazing	$f_{\text{no contact}}$	$h_{\text{near grazing}}$	g_{id}

Where:

$$
\begin{aligned}
h_{\text{contact}}(v,\alpha) &:= (x-\varepsilon_x)^2 + (y-\varepsilon_y)^2 - 1 = 0,\\
h_{2\pi}(v,\alpha) &:= s - 2\pi = 0,\\
h_{\text{near grazing}}(v,\alpha) &:= z = x' = 0,\\
g_{\text{id}}(v) &:= v,\\
g_{2\pi}(v) &:= (x, y, z, w, s - 2\pi).
\end{aligned}
$$

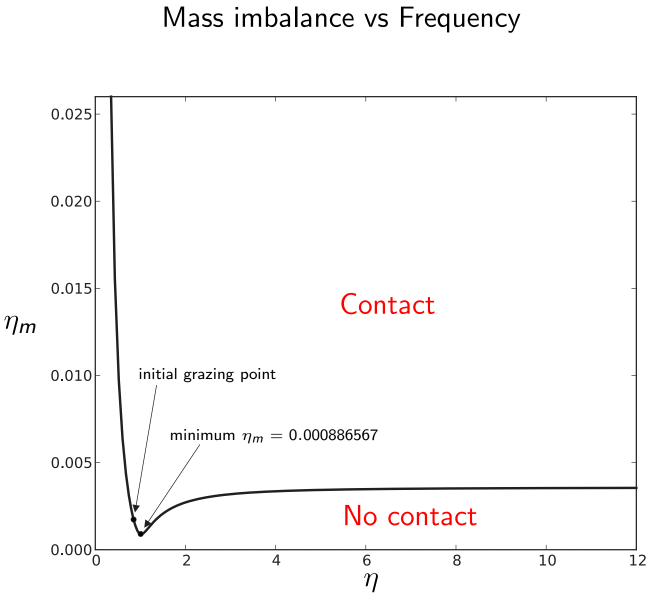
Bifurcation analysis with TC-HAT

Grazing orbit found during continuation at $\eta \approx 0.85281$ for $\eta_m = 0.0017$, $\rho = 70$, $\nu = 0.125$, $K = 30$, $\varepsilon_x = 0.75$ and $\varepsilon_y = 0$ fixed.



Bifurcation analysis with TC-HAT

Two-parameter continuation of the grazing curve in the η-η_m plane



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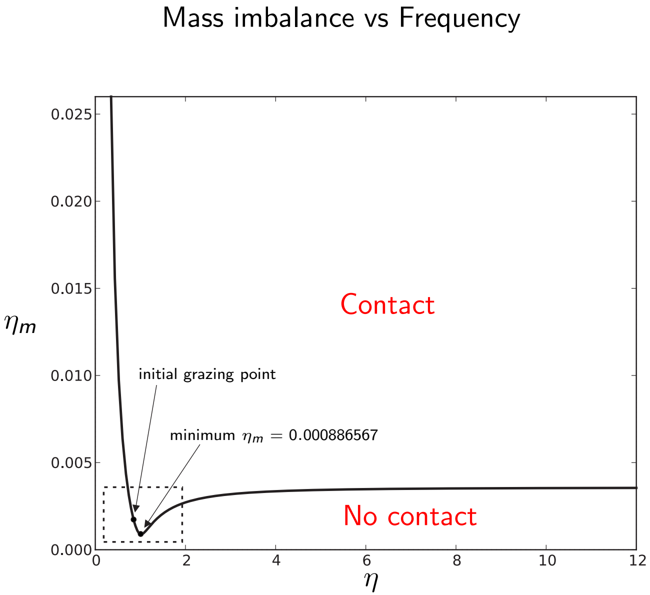
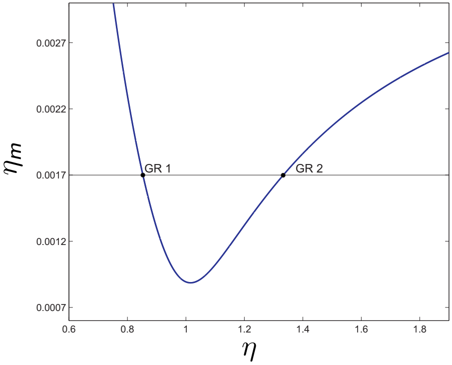
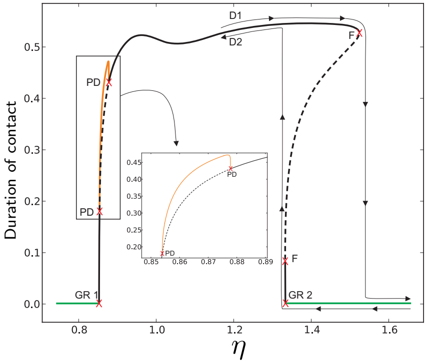


Bifurcation analysis with TC-HAT

Grazing curve

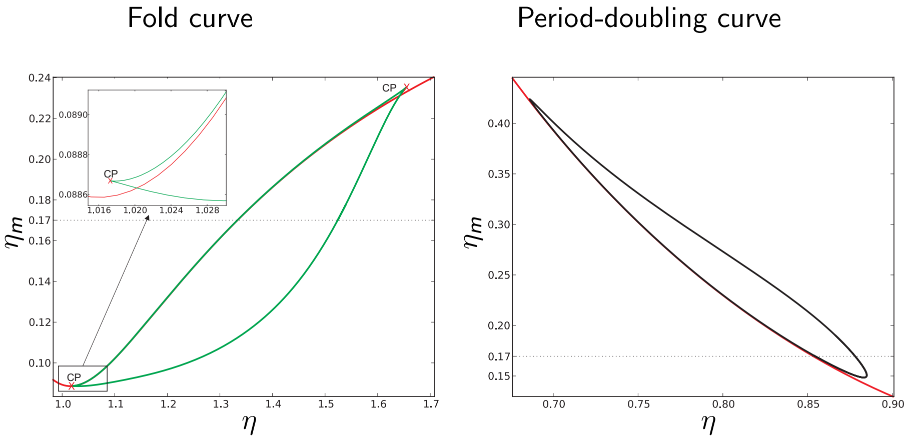


One-parameter continuation w.r.t. η



Bifurcation analysis with TC-HAT

Two-parameter continuation in the η-η_m plane (grazing curve in red)



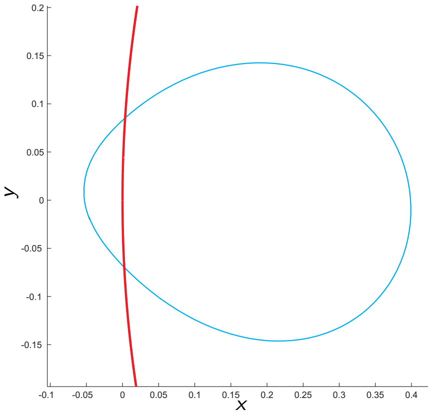
The values on the η_m-axis are multiplied by the factor 10^{-2}.

Chaotic Regime

For $\eta_m = 0.0017$, $\rho = 70$, $\nu = 0.11$, $K = 30$, $\varepsilon_x = 1$ and $\varepsilon_y = 0$, cf. Karpenko et al., Chaos, Solitons & Fractals, 2002.

Bifurcation analysis with TC-HAT

Initial **period-1** orbit (stable) for $\eta = 2.4$



The continuation task reveals a torus bifurcation at $\eta \approx 2.45989$ and a period-doubling at $\eta \approx 2.55$.

Bifurcation analysis with TC-HAT

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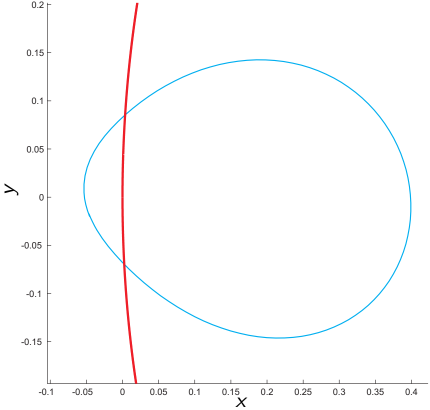
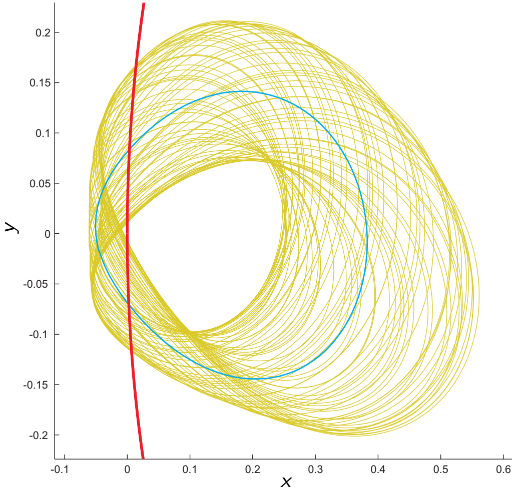


The continuation task reveals a torus bifurcation at $\eta \approx 2.45989$ and a period-doubling at $\eta \approx 2.55$.

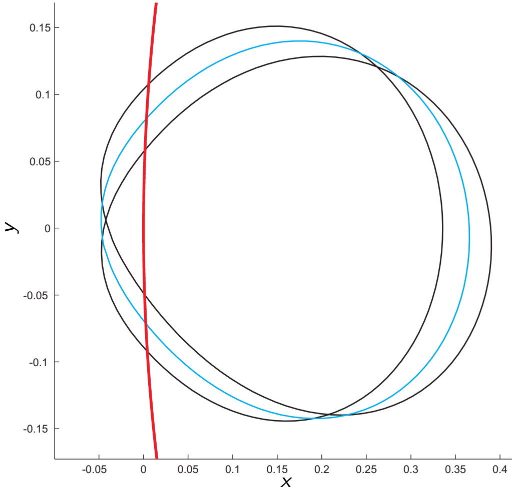
Bifurcation analysis with TC-HAT

Period-1 orbit (unstable) and **quasi-periodic** motion (stable) after the torus bifurcation, for $\eta = 2.47$.



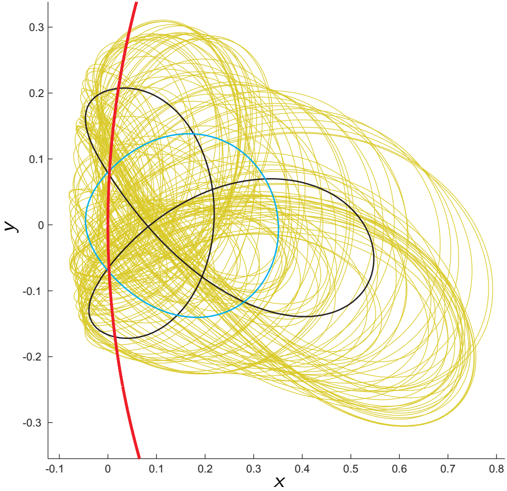
Bifurcation analysis with TC-HAT

Period-1 (unstable) and **period-2** orbits (unstable) after the PD bifurcation, for $\eta = 2.55128$.



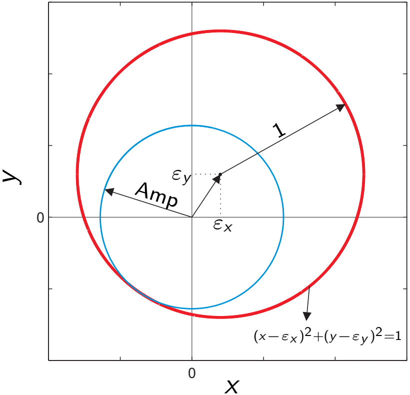
Bifurcation analysis with TC-HAT

Coexisting **period-1**, **period-2** (both unstable) and **chaotic motion** (stable) for $\eta = 2.6$.



Analytical grazing curve

Relation between frequency (η) and mass imbalance (η_m) producing grazing contact



Where $\text{Amp}(\eta_m, \eta, \rho, \nu) = \eta_m \sqrt{A(\eta, \rho, \nu)^2 + B(\eta, \rho, \nu)^2}$ and $A(\eta, \rho, \nu) = \dfrac{\rho\eta^2(1-\eta^2)}{4\nu^2\eta^2 + (1-\eta^2)^2}$, $B(\eta, \rho, \nu) = \dfrac{2\nu\rho\eta^3}{4\nu^2\eta^2 + (1-\eta^2)^2}$. According to the geometry of the system, grazing contact occurs when

$$\boxed{\text{Amp}(\eta_m, \eta, \rho, \nu) = 1 - \sqrt{\varepsilon_x^2 + \varepsilon_y^2}}$$

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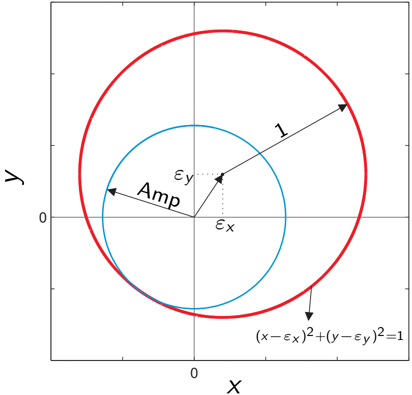

Where $\mathrm{Amp}(\eta_m, \eta, \rho, \nu) = \eta_m \sqrt{A(\eta, \rho, \nu)^2 + B(\eta, \rho, \nu)^2}$ and $A(\eta, \rho, \nu) = \dfrac{\rho\eta^2(1-\eta^2)}{4\nu^2\eta^2 + (1-\eta^2)^2}$, $B(\eta, \rho, \nu) = \dfrac{2\nu\rho\eta^3}{4\nu^2\eta^2 + (1-\eta^2)^2}$. According to the geometry of the system, grazing contact occurs when

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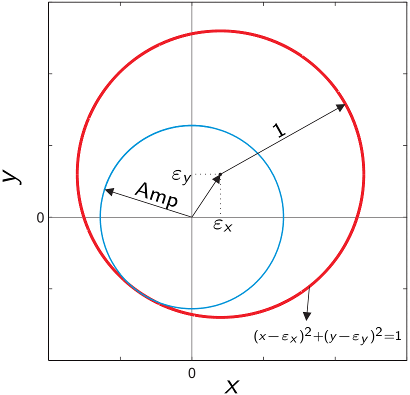

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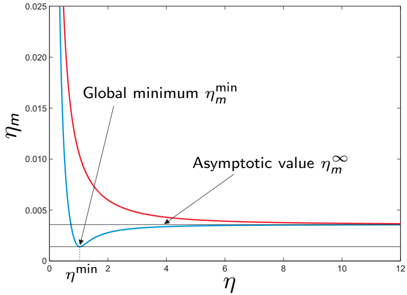
Analytical grazing curve

Relation between frequency (η) and mass imbalance (η_m) producing grazing contact

From the last condition we obtain the grazing curve:

$$\eta_m = G(\eta,\rho,\nu,\varepsilon_x,\varepsilon_y) = \frac{1-\sqrt{\varepsilon_x^2+\varepsilon_y^2}}{\sqrt{A(\eta,\rho,\nu)^2+B(\eta,\rho,\nu)^2}}$$



$$\eta_m^{\infty} = \lim_{\eta\to\infty} G(\eta,\rho,\nu,\varepsilon_x,\varepsilon_y) = \frac{1-\sqrt{\varepsilon_x^2+\varepsilon_y^2}}{\rho}$$

$$\eta^{min} = \frac{1}{\sqrt{1-2\nu^2}}, \text{ for } 0<\nu<\frac{1}{\sqrt{2}}$$

$$\eta_m^{min} = \frac{2\nu\left(1-\sqrt{\varepsilon_x^2+\varepsilon_y^2}\right)\sqrt{1-\nu^2}}{\rho}$$

Analytical grazing curve

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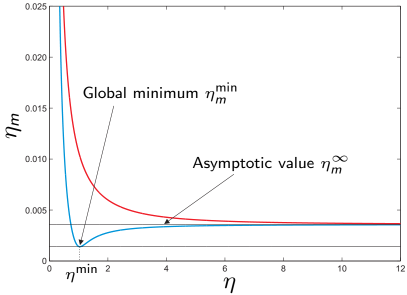

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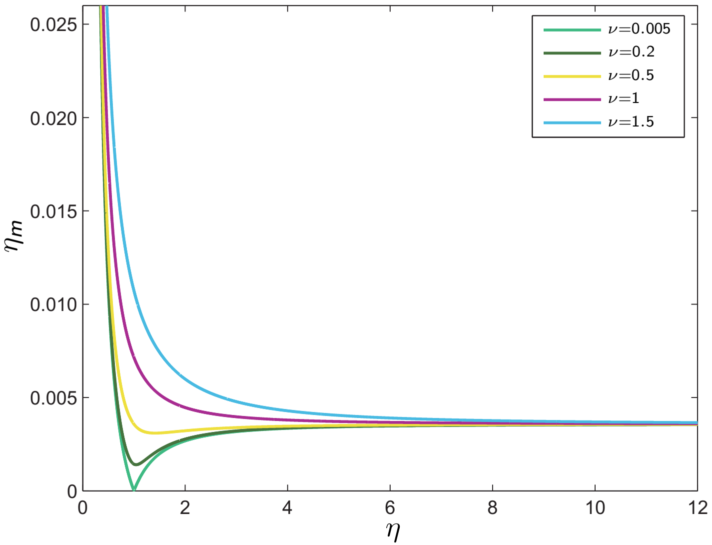
Analytical grazing curve

For different values of viscous damping



Future work

Experimental verification of the bifurcation scenarios

Rotor in action!

(Thanks to Vahid Vaziri, University of Aberdeen)

Acknowledgements

The speaker wishes to thank:

- Centre for Applied Dynamics Research, University of Aberdeen, Scotland,
- Organizers of the Modern Practice in Stress and Vibration Analysis Conference (MPSVA 2012), and
- The Audience!

Thanks a lot for your attention!