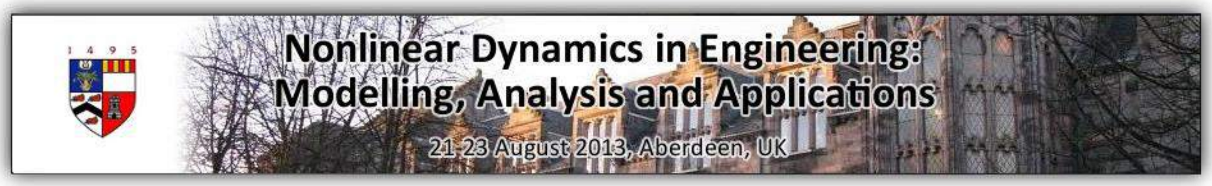

Book of Abstracts

Edited by
James Ing, Yang Liu, Ekaterina Pavlovskaia, Andrey Postnikov, Marian Wiercigroch

Aberdeen, August 2013

Contents

Preface

This international conference has been organised to mark 10th anniversary of the Centre for Applied Dynamics Research (CADR) which was founded in 2003. Since that time, CADR has grown into a multi-disciplinary research group with a strong focus on the application of dynamical systems theory to science and engineering. It aims to facilitate and enhance interactions among applied mathematicians, scientists, engineers and medical researchers. It is known for its harmonious blend of theoretical and experimentally rich research in a broad spectrum of dynamics including smooth and non-smooth dynamical systems, elastic stability, parametric and nonlinear oscillations, chaos control, classical and quantum relativity, bifurcations, transients and crisis, vibro-impact dynamics, vibration control, nonlinear time series analysis, synchronization, mechanics of supercoiling of DNA, molecular dynamic simulation, percussion drilling, underwater acoustics, spacio-temporal systems, condition monitoring, renewable energy, MEMS, neural networks and neural dynamics, and experimental methods.

The main aim of the conference is to critically assess the state-of-the-art of numerical, analytical and experiments methods applied to mechanics and nonlinear dynamics of all branches of engineering. The conference will gather leading international experts to review the state-of-the art, to outline future research directions and to stimulate development of new generation numerical, analytical and experimental techniques, materials, products and processes operating on principles of nonlinear dynamics. The meeting will focus on invited presentations and discussions with interactions among attendees to facilitate exchanges of ideas.

Over its 10 year history, CADR has welcomed more than 150 visitors and collaborators from around the World. In honour of the strong collaborations which have greatly contributed to the success of CADR, each of the sessions is named after a country or region with which we have a strong collaborative link. Let this conference be a celebration of the success of these connections.

Organisers & Contact

Chairs

Marian Wiercigroch, University of Aberdeen
Ekaterina Pavlovskaia, University of Aberdeen

Scientific Committee

Soumitro Banerjee, IISER Kolkata
Matthew Cartmell, University of Sheffield
Emmanuel Detournay, University of Minnesota
Tomasz Kapitaniak, Technical University of Lodz
Anton Krivtsov, IPME RAS St.Petersburg
Andrew Leung, City University of Hong Kong
Ekaterina Pavlovskaia, University of Aberdeen
Marcelo Savi, COPE, Rio de Janeiro
Geoffrey Tomlinson, University of Sheffield
Jerzy Warminski, Lublin University of Technology
Ko-Choong Woo, University of Nottingham
Qingjie Cao, Harbin Institute of Technology
Alan Champneys, University of Bristol
John Hogan, University of Bristol
Edwin Kreuzer, Hamburg University of Technology
Stefano Lenci, Marche Polytechnic University
Carlos Mazzilli, University of São Paulo
Giuseppe Rega, University of Rome "La Sapienza"
Michael Thompson, University of Aberdeen
Yoshisuke Ueda, Kyoto University
Marian Wiercigroch, University of Aberdeen
Hiroshi Yabuno, Keio University

Local Organising Committee

Olusegun Ajibose
Yang Liu
Andrey Postnikov
James Ing
Ekaterina Pavlovskaia (Co-Chair)
Marian Wiercigroch (Chair)

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Technical Programme

Day 1: 21 August 2013

UK Session
Chair: Marian Wiercigroch, University of Aberdeen

8:00 – 8:20 Opening Ceremony: **Marian Wiercigroch**, University of Aberdeen; **Bryan MacGregor**, University of Aberdeen; **John Reynolds**, Aberdeen City Council

8:20 – 9:00 Opening Lecture: **Alan Champneys**, University of Bristol, *The dynamics of bouncing, squeaking and rattling*

9:00 – 9:20 Contributing Lecture: **Viktor Avrutin**, University of Stuttgart, *Bifurcations of chaotic attractors in 1D piecewise-smooth maps: current state and open problems*

9:20 – 9:40 Contributing Lecture: **Qishao Lu**, BUAA, *Synchronization and patterns of a noisy clustered neuronal network*

9:40 – 10:00 Contributing Lecture: **Petri Piiroinen**, NUI Galway, *Dynamics of gears with impacts*

10:00 – 10:20 Contributing Lecture: **Alexandre Depouhon**, Université de Liège, University of Minnesota, *On a modified bilinear law to model bit/rock interaction in percussive drilling*

10:20 – 10:40 **Coffee break**

Italian Session
Chair: Stefano Lenci, Polytechnic University of Marche

10:40 – 11:00 Contributing Lecture: **Mike Jeffrey**, University of Bristol, *Probing the realism of nonsmooth dynamics*

11:00 – 11:20 Contributing Lecture: **Andrzej Stefanski**, Lodz University of Technology, *Synchrony in systems with friction*

11:20 – 11:40 Contributing Lecture: **Pankaj Wahi**, IIT Kanpur, *An acceleration-dependent friction model suitable for friction-induced vibration studies*

11:40 – 12:00 Contributing Lecture: **Qingjie Cao**, Harbin Insitute of Technology, *Nonlinear dynamics of an irrational oscillator*

12:00 – 12:20 Contributing Lecture: **Joseph Paez**, University of Aberdeen & ESPOL, *Bifurcation study of a vibro-impact oscillator with drift*

12:20 – 12:40 Contributing Lecture: **Yoshisuke Ueda**, Kyoto University, *Cusp of a cone opening upward seemed to be an attracting launcher under horizontal periodic forcing: occurrence of intermittently explosive chaotic motion*

12.40 – 14.00 **Lunch Break & European Poster Session**

Chair: Andrzej Stefanski, Lodz University of Technology

Olusegun Ajibose, University of Aberdeen, *Finite element modelling of rotary impact drilling*

Antonio Chong, University of Aberdeen, *Computational methods for non-smooth dynamical systems*

Anindya Chatterjee, IIT Kanpur, *Modal damping prediction for vibrating solids: constitutive models and finite element computations*

Ying Du, East China University of Science and Technology, *Spike train pattern in a network model of the olfactory bulb*

Shihui Fu, Zhengzhou University, *Bifurcation analysis in n-scroll modified Chua's circuit*

Marcin Kapitaniak, University of Aberdeen, *Dynamics of drill-strings with Cosserat rod theory*

Marko Keber, University of Aberdeen, *Influence of tension variation on VIV: a preliminary study*

Krzysztof Kecik, Lublin University of Technology, *Dynamics of an active autoparmetric system*

Lili Laczak, Budapest University of Technology and Economics, *Aircraft impact into reinforced concrete structure*

Maolin Liao, University of Aberdeen, *Drill-bit - formation interaction investigated using an impact oscillator*

Everton Medeiros, University of Sao Paulo, *Torsion properties of periodic states*

Marina Menshykova, University of Aberdeen, *Nonlinear contact problems in cracks' dynamics*

Joseph Paez, University of Aberdeen & ESPOL, *Experimental study of rotor vibrations*

Mikhail V. Zakrzhevsky, Riga Technical University, *Application of the bifurcation theory of nonlinear dynamical systems and new rare attractors*

Chinese Session
Chair: Andrew Leung, City University of Hong Kong

14:00 – 14:40 Keynote Lecture: **Marcelo Savi**, Federal University of Rio de Janeiro, *Nonlinear dynamics, chaos and control of smart material systems*

14:40 – 15:00 Contributing Lecture: **Anindya Chatterjee**, IIT Kanpur, *Reduced order models from high dimensional frictional hysteresis*

15:00 – 15:20 Contributing Lecture: **Ko-Choong Woo**, University of Nottingham Malaysia Campus, *Rotating a pendulum with a nonlinear excitation*

15:20 – 15:40 Contributing Lecture: **Piotr Omenzetter**, University of Aberdeen, *Evaluation of nonlinear seismic responses of full-scale instrumented buildings*

15:40 – 16:00 Contributing Lecture: **Janko Slavic**, University of Lubjlana, *Frequency-domain methods for vibration-fatigue-life estimation*

16:00 – 16:20 **Coffee break**

Industrial Session
Chair: Ekaterina Pavlovskaia, University of Aberdeen

16:20 – 17:00 Keynote Lecture: **Patrick O'Brien**, ITF, *Nonlinear mechanics and dynamics challenges for Subsea pipelines and risers*

17:00 – 17:20 Contributing Lecture: **Xu Xu**, GE, *Global workover riser analysis*

17:20 – 17:40 Contributing Lecture: **Adrian Connaire**, MCS Kenny, *Quasi-rotation method for beams undergoing large deflection with coupled torsion, bending and axial deformation*

19.00 for 19.30 **Civic Reception**

Day 2: 22 August 2013

Brazilian Session
Chair: Qishao Lu, BUAA

8:00 – 8:40 Keynote Lecture: **Giuseppe Rega**, University of Rome 'La Sapienza', *Nonlinear dynamics of atomic force microscopy*

8:40 – 9:00 Contributing Lecture: **Jan Sieber**, University of Exeter, *Using feedback control for discovery*

9:00 – 9:20 Contributing Lecture: **Paulo Gonçalves**, Catholic University of Rio de Janeiro, *The influence of imperfections and uncertainties in nonlinear structural dynamics*

9:20 – 9:40 Contributing Lecture: **Fang Han**, Donghua University, *Phase synchronization of coupled small-world neuronal networks with short-term synaptic plasticity*

9:40 – 10:00 Contributing Lecture: **Kiyotaka Yamashita**, Fukui University of Technology, *Nonlinear interactions between unstable vibartion modes of a fluid-conveying pipe*

10:00 – 10:20 Contributing Lecture: **Oded Gottlieb**, Technion, *Nonlinear dynamics of thermo-visco-elastic cantilever sensor arrays*

10:20 – 10:40 **Coffee break**

Indian Session
Chair: Christophe Pierre, University of Illinois

10:40 – 11:00 Contributing Lecture: **Edwin Kreuzer**, TUHH, *Dynamic analysis of tank container handling with arbitrary filling level*

11:00 – 11:20 Contributing Lecture: **György Károlyi**, Budapest University of Technology and Economics, *Stress-free layers in photoinduced deformations of beams*

11:20 – 11:40 Contributing Lecture: **Francesco Romeo**, University of Rome 'La Sapienza', *Nonlinear dynamics of a mechanical oscillator coupled to an electro-magnetic circuit*

11:40 – 12:00 Contributing Lecture: **Przemyslaw Perlikowski**, Lodz University of Technology, *Numerical optimization of Duffing oscillator*

12:00 – 12:20 Contributing Lecture: **Yuichi Yokoi**, Nagasaki University, *Rotation disturbed by second harmonic excitation in parametrically excited pendulum*

12:20 – 12:40 Contributing Lecture: **David Wagg**, University of Bristol, *A new normal form method for analysing systems of coupled nonlinear oscillators*

13:00 – 19.00 Castle Trip

19:00 Conference Dinner

Day 3: 23 August 2013

European Session
Chair: Edwin Kreuzer, TUHH

8:00 – 8:40 Keynote Lecture: **Tomasz Kapitaniak**, Lodz University of Technology, *Synchronous states of slowly rotating pendula*

8:40 – 9:00 Contributing Lecture: **Mike Graham**, Imperial College London, *Non-linear viscous damping of floating bodies*

9:00 – 9:20 Contributing Lecture: **Dmitry Indeitsev**, Russian Academy of Science, *Localized modes in one-dimensional continuous structures of finite and infinite length*

9:20 – 9:40 Contributing Lecture: **Jason Reese**, University of Strathclyde, *Molecular flow engineering*

9:40 – 10:00 Contributing Lecture: **Davide Dionisi**, University of Aberdeen, *Effect of mesh size and spatial discretisation schemes on the solution of turbulent flow equations in agitated vessels*

10:00 – 10:20 Contributing Lecture: **Paul Manneville**, Ecole Polytechnique, *Transitional wall-bounded flow: where do we stand?*

10:20 – 10:40 **Coffee break**

American Session
Chair: Jason Reese, University of Strathclyde

10:40 – 11:00 Contributing Lecture: **Nikita Morozov**, Russian Academy of Sciences, *The rod dynamics under longitudinal impact*

11:00 – 11:20 Contributing Lecture: **Stefano Lenci**, Polytechnic University of Marche, *Nonlinear vibration of a microbeam modeled by the strain gradient elasticity with an electrical actuation approximated by the Padé-Chebyshev method*

11:20 – 11:40 Contributing Lecture: **Mathias Legrand**, McGill University, *Rotor/stator interaction within a helicopter engine*

11:40 – 12:00 Contributing Lecture: **Caishan Liu**, Peking University, *Periodic behaviours in a 3D nonlinear system*

12:00 – 12:20 Contributing Lecture: **Silvio de Souza**, University of Ouro Preto, *Self-similarities of periodic structures for autoparametric oscillators*

12:20 – 12:40 Contributing Lecture: **Jerzy Warminski**, Lublin University of Technology, *Bifurcations in frictional model of cutting process*

12:20 – 14.00 **Lunch Break & Asian Poster Session**

Chair: Jerzy Warminski, Lublin University of Technology

Shane Burns, National University of Ireland, Galway, *Impacts with friction*

Si-Chung Jong, University of Nottingham, *Nonlinear dynamics of a vibro-impact machine subjected to electromagnetic interactions*

Yang Liu, University of Aberdeen, *Analysis and control of an underactuated drill-string*

Debshankha Manik, Indian Institute of Science, *Education and Research, Kolkata, Impact-induced transients close to grazing in an impact oscillator*

Richard Morrison, University of Aberdeen, *Stability analysis of a two pendulum system*

Anna Najdecka, University of Aberdeen, *Rotational motion of a parametric pendulum excited on a plane*

Andrey Postnikov, University of Aberdeen, *An approach to calibration of low dimensional VIV models using CFD*

Ashesh Saha, IIT Kanpur, *Characterisation of friction force and nature of bifurcation from experiments on a single-degree-of-freedom friction-induced system*

Mukthar Sayah, University of Aberdeen, *Attractor reconstruction for parameter identification in an impact oscillator*

Vahid Vaziri, University of Aberdeen, *Experimental study of a drill-string assembly*

Vahid Vaziri, University of Aberdeen, *Experimental study of control methods for maintaining rotation of parametric pendulum*

Zhiying Qin, Hebei University of Science and Technology, *Dynamic model and analysis of non-harmonic vibration of conveyor*

Yao Yan, University of Aberdeen, *Bifurcation and quench control of grinding chatter*

Zhuoqin Yang, Beihang University, *Dynamics of compound bursting composed of different bursts and subthreshold oscillation*

Qi Xu, Nanjing University of Aeronautics and Astronautics, *A nonlinear model of balancing human standing*

Yusheng Zhou, Nanjing University of Aeronautics and Astronautics, *Design of the delayed optimal feedback control for linear systems with multiple delayed inputs*

Russian Session
Chair: Mike Graham, Imperial College London

14:00 – 14:20 Contributing Lecture: **Andrew Leung**, City University of Hong Kong, *Periodic bifurcation analysis of fractional derivative and delay systems*

14:20 – 14:40 Contributing Lecture: **Oleg Gendelman**, Technion, *Exact solutions for discrete breathers in forced - damped chain*

14:40 – 15:00 Contributing Lecture: **James Ing**, University of Aberdeen, *Stability analysis for intermittent control of co-existing attractors*

15:00 – 15:20 Contributing Lecture: **Hong Ling**, Xi'an Jiaotong University, *Crisis in chaotic pendulum with fuzzy uncertainty*

15:20 – 15:40 Contributing Lecture: **Yuri Miklin**, Kharkov Polytechnic University, *Forced nonlinear normal modes in one disk rotor dynamics*

15:40 – 16:00 Contributing Lecture: **Ron Chen**, City University of Hong Kong, *Constructing simple chaotic systems with an arbitrary number of equilibria or of scrolls*

16:00 – 16:20 **Coffee break**

Polish Session
Chair: Ron Chen, City University of Hong Kong

16:20 – 16:40 Contributing Lecture: **Yang Liu**, University of Aberdeen, *Forward and backward motion control of a vibro-impact capsule system*

16:40 – 17:00 Contributing Lecture: **Zi-Qiang Lang**, University of Sheffield, *Monitoring of the characteristic parameter changes of a nonlinear oscillator by nonlinear system modelling and analysis*

17:00 – 17:20 Contributing Lecture: **Murilo Baptista**, University of Aberdeen, *How common periodic stable behavior appears in nonlinear dissipative (mechanical) systems*

17:20 – 17:40 Contributing Lecture: **Sergey Kryzhevich**, St Petersburg State University, *One-dimensional chaos in a system with dry friction*

17:40 – 18:20 Closing Lecture: **Michael Thompson**, University of Cambridge, *Predicting the approach to an oscillatory instability at a Hopf bifurcation*

18:20 Closing Ceremony: **Marian Wiercigroch**, University of Aberdeen

Social Programme

Welcome to Aberdeen and the region — home to the best whisky, the best food and the highest concentration of castles in the UK.

Scottish hospitality is World renowned and this conference will make no exception. The social programme manages to pack a range of cultural experiences into a very short time to give a taste of the best that Aberdeenshire has to offer.

20 August 2013 - Welcome Drinks Reception (Fraser Noble Building Foyer) 6pm - 9pm

Enjoy a glass of wine or an orange juice while meeting colleagues before the official start of the conference. The Fraser Noble Building is located in the King's College Campus, no. 7 on the campus map.

21 August 2013 - Civic Reception (Aberdeen Town House) 7pm for 7.30pm

The historic Townhouse located in the Castlegate area of the city centre will be the venue for the civic reception, kindly sponsored by Aberdeen City Council.

22 August 2013 - Castle-Whisky Tour (Fyvie Castle and Glen Garioch Distillery) 1pm – 7pm & Conference Dinner (Elphinstone Hall) commencing at 7pm

On the afternoon of the 22nd there will be an excursion into Aberdeenshire in order to soak up some of the rich history of the region. Visit the World famous 'haunted' Castle at Fyvie on the banks of the river Ythan. A packed lunch will be provided. Following this is a visit to Glen Garioch Distillery. Located in the town of Oldmeldrum, the distillery has been operating since 1797 and is the Country's most easterly distillery.

In the evening enjoy the conference dinner, with a selection of fresh local cuisine. Elphinstone Hall is no. 25 on the campus map.

Post-conference events

The 172nd annual Lonach Highland Gathering and Games will take place on Saturday 24th August 2013 in the village of Bellabeg, Strathdon which is approximately 42 miles from Aberdeen. For other post-conference events such as local tours please check VisitScotland and ScotlandExplorer.

Abstracts

Day 1: 21 August 2013

UK Session (8:00 – 10:20)

"The dynamics of bouncing, squeaking and rattling", A Champneys

"Bifurcations of chaotic attractors in 1D piecewise-smooth maps: current state and open problems", V Avrutin, L Gardini, I Sushko

"Synchronization and patterns of a noisy clustered neuronal network", Q Lu, X Sun

"Dynamics of gears with impacts", P T Piiroinen, J Mason

"On a modified bilinear law to model bit/rock interaction in percussive drilling", A Depouhon, V Denoël, E Detournay

Italian Session (10:40 – 12:40)

"Probing the realism of nonsmooth dynamics", M R Jeffrey

"Synchrony in systems with friction", A Stefanski, J Wojewoda, M Wiercigroch, T Kapitaniak

"An acceleration-dependent friction model suitable for friction-induced vibration studies", A Parikh, P Wahi

"Nonlinear dynamics of an irrational oscillator", Q Cao, Y Xiong, M Wiercigroch

"Bifurcation study of a vibro-impact oscillator with drift", J Paez, M Wiercigroch, E Pavlovskaia

"Cusp of a cone opening upward seemed to be an attracting launcher under horizontal periodic forcing: occurrence of intermittently explosive chaotic motions", Y Ueda

European Poster Session (12:40 – 14:00)

"Finite element modelling of rotary impact drilling", O Ajibose, M Wiercigroch, A R Akisanya

"Computational methods for non-smooth dynamical systems", A Chong, J Paez, M Wiercigroch

"Modal damping prediction for vibrating solids: constitutive models and finite element computations", P Jana, A Chatterjee

"Spike train pattern in a network model of the olfactory bulb", Y Du, R Wang, Q Lu

"Bifurcation analysis in n-scroll modified Chua's circuit", S H Fu, Q S Lu

"Dynamics of drill-strings with Cosserat rod theory", M Kapitaniak, M Wiercigroch

"Influence of tension variation on VIV: a preliminary study", M Keber, J Warminski, M Wiercigroch

"Dynamics of an active autoparmetric system", K Kecik, R Rusinek, J Warminski, M Wiercigroch

"Aircraft impact into reinforced concrete structure", L E Laczak, Gy Károlyi

"Drill-bit - formation interaction investigated using an impact oscillator", M Liao, J Ing, J Paez Chavez, M Wiercigroch

"Torsion properties of periodic states", E Medeiros, R O Medrano-T, I L Caldas, S L T de Souza

"Nonlinear contact problems in cracks' dynamics", M Menshykova, O Menshykov

"Experimental study of rotor vibrations", J Paez Chavez, V Vaziri, M Wiercigroch

"Application of the bifurcation theory of nonlinear dynamical systems and new rare Attractors, M V Zakrzhevsky

Chinese Session (14:00 – 16:00)

"Nonlinear dynamics, chaos and control of smart material systems", M Savi

"Reduced order models from high dimensional frictional hysteresis", S Biswas, A Chatterjee

"Rotating a pendulum with a nonlinear excitation", S-H Teh, K-H Chan, K-C Woo, H Demrdash

"Evaluation of nonlinear seismic responses of full-scale instrumented buildings", F Butt, P Omenzetter

"Frequency-domain methods for vibration-fatigue-life estimation", J Slavič, M Mršnik, M Boltežar

Industrial Session (16:20 – 17:40)

"Nonlinear mechanics and dynamics challenges for Subsea pipelines and risers", P O'Brien

"Global workover riser analysis", X Xu, H Hashemizadeh, G Morgan

"Quasi-rotation method for beams undergoing large deflection with coupled torsion, bending and axial deformation", A Connaire, A Harte, P O'Brien

THE DYNAMICS OF BOUNCING, SQUEAKING AND RATTLING

Alan Champneys
Department of Engineering Mathematics and Head, Queen's School of Engineering, University of Bristol, Bristol BS8 1TR, UK
a.r.champneys@bristol.ac.uk

ABSTRACT

In this talk I shall introduce recent work on several applied problems that use the theory of bifurations in piecewise smooth dynamical systems that I have developed over the years in collaboration with Chris Budd, Mario Di Bernardo and Piotr Kowalczyk [1] and with many others. The first problem, in collaboration with Arne Nordmark and Harry Dankowicz [2] will involve why you can drag a piece of chalk but not push it. This problem is known as the Painleve paradox. The second problem which involves Csaba Hos and Csaba Bazso (an extension of the results in [3]) concerns pressure valve chatter helped me explain why my office radiator developed an annoying rattle. The third problem is the so-called two-ball bounce problem and involves what happens when you drop to the floor a lighter ball resting on top of a heavier elastic ball; which is ongoing work with my PhD student Yani Berdini, co-supervised by Robert Szalai.

Each of the three examples can be modelled using a low-order nonsmooth dynamical system formulation. But in each case there is extra complexity that goes beyond the standard theory of piecewise-smooth systems as reviewed in [1].

The first example involves an understanding of impact with friction, which using a rigid impact and dry friction leads to inevitable indeterminate situations. We show one of the consequences of this is the possible excitation of "reverse chatter" where a sequence of impacts accumulates backwards in time. It is argued that this kind of motion underlies the juddering motion of chalk when it is attempted to be pushed rather than dragged.

The second example involves non-smooth dynamics due to valve impact that is further complicated by the presence of flow induced oscillation. An interaction is found to occur between several different bifurcations, both smooth and non-smooth which leads to a complex bifurcation diagram involving bi-stability of smooth and non-smooth orbits, together with a Hopf-Hopf codimension two-point that occurs very close to the grazing bifurcation.

The final example is simple to solve in the framework of rigid body dynamics if it is assumed that the impacts occur in a certain order. We have conducted detailed experiments to quantify the effect, and it is found that there is another regime where rigid-body dynamics does not seem to apply. Instead we have to build up a continuum theory based on wave propagation and refocusing in the lower ball.

References

[1] M. di Bernardo, C. Budd, A. Champneys and P. Kowalczyk: Piecewise-smooth dynamical systems: theory and applications. *Springer-Verlag* (2008).
[2] A. Nordmark, H. Dankowicz and A. Champneys: Friction-induced reverse chatter in rigid-body mechanisms with impact. *IMA Journal of Applied Mathematics* **76** (2011) 85-119.
[3] C. Hos and A. Champneys: Grazing bifurcations in a pressure relief valve model. *Physica D* **241** (2012) 2068-2076.

BIFURCATIONS OF CHAOTIC ATTRACTORS IN 1D PIECEWISE-SMOOTH MAPS: CURRENT STATE AND OPEN PROBLEMS

V. Avrutin[1], L.Gardini[2], I.Sushko[3]
[1]*DEPS, University of Urbino, Italy and IST, University of Stuttgart, Germany*
Avrutin.Viktor@gmail.com
[2]*DEPS, University of Urbino, Italy*
Laura.Gardini@uniurb.it
[3]*Institute of Mathematics, National Academy of Sciences of Ukraine, Ukraine*
Sushko@imath.kiev.ua

ABSTRACT

The theory of non-smooth dynamical systems, initially being a peripheral research area of Nonlinear Dynamics, represents nowadays a well established and rapidly growing domain, accepted by the scientific community both from the theoretical as well as from the practical point of view. In particular, it is known that smooth systems are not sufficient to provide an adequate description for many phenomena occurring in technical applications and the nature. Recently, more and more components of technical systems with increasing practical importance are proved to be adequately described by non-smooth systems. The most well-known examples of such systems belong to such fields as electronics (A/D converters and DC/DC power converters present nowadays in every computer, every mobile telephone, etc.) and mechanics (systems with impacts or friction). Other application examples are known in computer science (mobile communication systems, computer networks), control theory, economics and financial market modelling as well as in social sciences.

One of the significant differences between smooth and piecewise-smooth systems regards the persistence of chaotic attractors under parameter perturbations. It is known that in smooth maps in generic case chaotic attractors exist only on singular parameter values, while in piecewise-smooth maps chaos may exist in open sets in parameter space (in this case chaos is referred to as robust [1]). Therefore, when dealing with piecewise-smooth maps we have to consider extended regions in parameter space, in which all attractors are chaotic (so-called chaotic domain). In this domain several bifurcations may occur, at which the geometrical shape and typically the number of connected components (bands) of chaotic attractors changes. All these transitions are associated with *homoclinic bifurcations* of unstable cycles and can be classified according to their properties. Characteristic for these bifurcations is the presence of so-called *critical homoclinic orbits* to these cycles [2]. Among these bifurcations, three types are most relevant for 1D maps:

1. *Merging bifurcation* (also referred to as merging crisis or crisis-like transition), which occurs when the unstable cycle has a *negative* eigenvalue and is non-homoclinic before the bifurcation.
2. *Expansion bifurcation* (also referred to as interior crisis), which occurs when the unstable cycle has a *positive* eigenvalue and is either non-homoclinic or one-side homoclinic before the bifurcation.
3. *Final bifurcation* (also referred to as boundary crisis or exterior crisis), which is associated with the same conditions as an expansion bifurcation and differs from it only by the fact that after the bifurcation a chaotic attractor is transformed into a chaotic repeller.

Examples of merging, expansion and final bifurcations in the general piecewise-linear map given by

$$x_{n+1} = f(x_n) = \begin{cases} f_{\mathcal{L}}(x_n) &= a_{\mathcal{L}} x_n + \mu_{\mathcal{L}} & \text{if} \quad x_n < 0 \\ f_{\mathcal{R}}(x_n) &= a_{\mathcal{R}} x_n + \mu_{\mathcal{R}} & \text{if} \quad x_n > 0 \end{cases} \tag{1}$$

are shown in Figs. 1,2 and 3, respectively.

It is also known that sequences of merging and expansion bifurcations may form several generic bifurcation scenarios, as in particular the *bandcount doubling*, *bandcound adding* and *bandcount incrementing* cascades [3,4]. Possible effects of the bifurcations mentioned above as well as specific properties of the bifurcation scenarios formed by them depend strongly on the properties of the considered map. In particular, we have to distinguish between

1. maps with *one* border point and maps with *several* border points,
2. *continuous* maps (for which multi-band chaotic attractors are necessarily cyclic) and *discontinuous* maps (in which case multi-band chaotic attractors may also be acyclic, as shown in Fig. 4. Necessary and sufficient conditions for appearance of cyclic and acyclic chaotic attractors in discontinuous maps are reported in [5]).

The goal of the talk is to give an overview of the bifurcations and bifurcation scenarios occurring in the domain of robust chaos in 1D piecewise-smooth maps and to point out some challenging problems which appear naturally in this field but are currently still open.

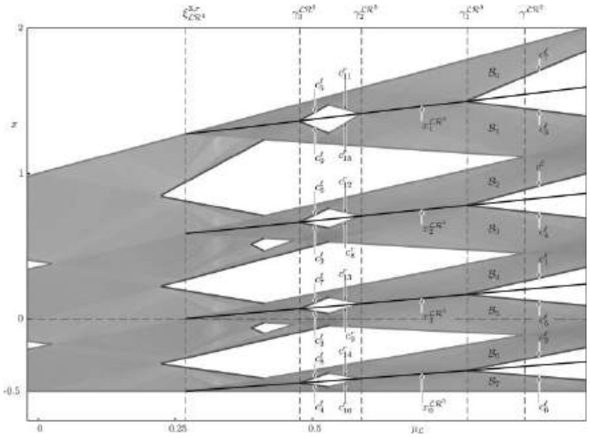

Figure 1: Examples for merging bifurcations in map (1), caused by homoclinic bifurcations of the 4-cycle LR^4

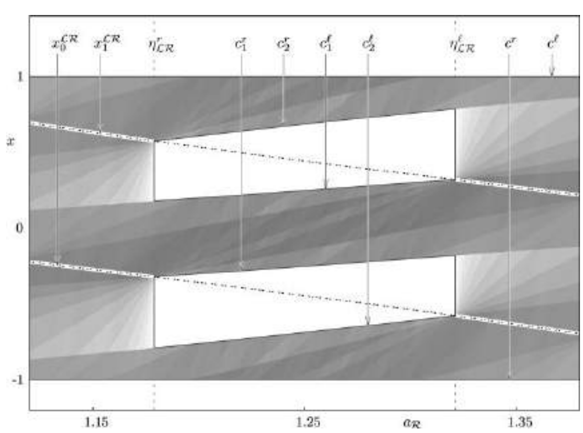

Figure 2: Examples for expansion bifurcations in map (1), caused by homoclinic bifurcations of the 2-cycle *LR*

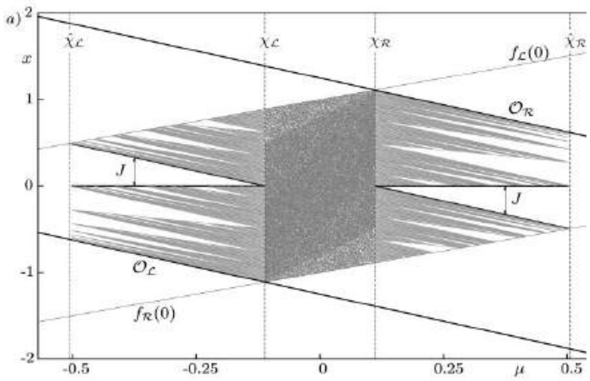

Figure 3: Examples for final bifurcations in map (1), caused by homoclinic bifurcations of the fixed points *L* and *R*

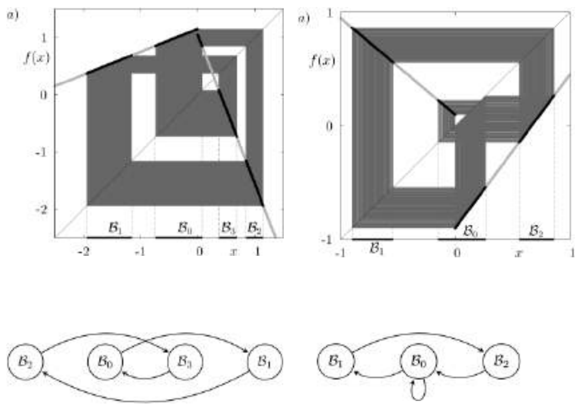

Figure 4: Cyclic (left) and acyclic (right) chaotic attractors in map (1).

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SYNCHRONIZATION AND PATTERNS OF A NOISY CLUSTERED NEURONAL NETWORK

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ABSTRACT

Synchronization is ubiquitous in nature and plays an important role in different situations, such as predator-prey process, birds migratory and heartbeats. Synchronization has also been experimentally observed in nervous systems, such as hippocampal CA1 area [1], retina [2] and striate cortex [3]. In the human brain, neurons in some communities usually behave synchronously to realize nervous information functions. Nevertheless, as we know, many of mental diseases, such as Parkinson's disease and epilepsy, are bound up with ideas of widespread rhythmic synchronization of neuronal elements. Thus, synchronization is essential in carrying out information functions but may be harmful sometimes in nervous systems.

It is commonly accepted that cortical architecture and connections are organized in a hierarchical and clustered (modular) way, called complex clustered network. Clustered structures have been found in several cortical systems and may be ideal for achieving high functional complexity [4-6].

Synchronization behavior is discussed here in a clustered neuronal network with additive noise applied. Neurons are distributed on a ring so that the sub-network is a regular one. In the neuronal network, N neurons of Rulkov's type are grouped into M clusters, and each cluster contains $k = N/M$ neurons. Each neuron connects to m nearest neighbors in the same cluster, and each pair of neurons in the nearest clusters is connected with a probability p. The mathematical model of the *i*-th neuron of the network is presented as follows:

$$\begin{cases} x_i(n+1) = \dfrac{\alpha}{1+x_i^2(n)} + y_i(n) + \eta_i(n) + \varepsilon \sum_{j=1}^{N} a(i,j)(x_j(n) - x_i(n)), \\ y_i(n+1) = y_i(n) - \beta x_i(n) - \gamma, \end{cases} \qquad (i = 1, \ldots, N,\ n = 1, 2, \ldots)$$

where $x_i(n)$ is the membrane potential of the neuron and $y_i(n)$ is the variation of the ion concentration. n is the iterated time index, ε is the coupling strength, and $a(i,j)=1$ if the *i*-th and *j*-th neurons are connected, otherwise $a(i,j)=0$. $\eta_i(n)$ is the additive noise.

Taking account of those changes in the coupling strength and the neuronal connection are closely interlaced with brain plasticity, the effects of the cluster number, the coupling strength and the noise intensity on synchronization and spatiotemporal patterns in the network are discussed in details. It is revealed that clustered structure of networks in noisy environments is able to make sub-networks more synchronous but suppress the synchrony of the entire network meanwhile. These results can help us to understand the existence of clustered structure in cortical systems from the viewpoint of synchronization.

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DYNAMICS OF GEARS WITH IMPACTS

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ABSTRACT

In a series of papers [1]-[5] the authors have used a variety of analytical and numerical techniques to analyse the local and global dynamics of a model of a pair of symmetric 1:1 meshing spur gears. The analyses have revealed a rich and complex dynamics where standard smooth local and global bifurcations interplay with grazing bifurcations. For the analyses the non-dimensional equation of motion

$$\Phi'' + \delta\Phi' = 4\pi\delta - 4\pi^2\varepsilon\cos(2\pi t) - 2\pi\delta\varepsilon\sin(2\pi t)$$

is used with perfectly elastic impacts at times t_{imp} so that

$$\Phi'(t^+_{imp}) = -\Phi'(t^-_{imp}) \quad \text{when} \quad \Phi(t^-_{\text{imp}}) = \pm\beta,$$

where $\Phi(t)$ denotes the relative rotational displacement at time t, δ is the non-dimensional damping coefficient, $2\beta > 0$ is the non-dimensional backlash width. Furthermore, ε is the non-dimensional amplitude related to the external forcing of the oscillation caused by eccentric mounting of the gears.

Most of the analyses are based on first-return maps, where one of the two boundaries is used as a Poincaré surface. The first-return maps are then used to locate periodic orbits of different itinerary, calculate one and two-parameter bifurcation diagrams using brute-force and continuation methods, determine domains of attraction using cell-to-cell mapping, continue stable and unstable manifolds and to describe the change in global dynamics due to interactions between global and grazing bifurcations.

Figure 1 depicts three typical graphical descriptions numerical analysis may give. Each description will reveal a certain part of the full picture and often there is a need to collect a variety of information in order to gain a greater collected understanding of the overall dynamics.

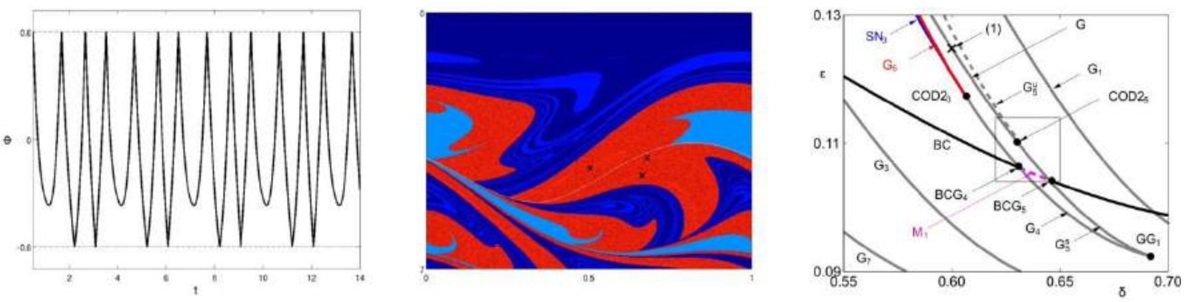

Figure 1: (left) A typical time history of the gear model showing a period-3 motion. (middle) An example of a domain of attraction figure with four different attractors. Each colour corresponds to an initial condition leading to a specific attractor. (right) An example of a two-parameter bifurcation diagram showing saddle-node bifurcations (SN), grazing bifurcations (G), boundary crises (BG), basin-boundary metamorphoses (M) as well as the codimension-two bifurcations grazing-grazing (GG), boundary-crisis-grazing (BSC) and others (COD2).

Here we will give a historical overview of the different numerical analysis techniques used. Some of the earlier results, as seen in Figure 1, will be described and a few new results relating to the interactions between unstable manifolds and discontinuity surfaces will be given. Finally we will also discuss future developments, possibilities and open questions.

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ON A MODIFIED BILINEAR LAW TO MODEL BIT/ROCK INTERACTION IN PERCUSSIVE DRILLING

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ABSTRACT

Experiments on dynamic indentation of rocks have revealed the sequential and rate-independent nature of bit/rock interaction during a single impact [1]. The force/penetration curve consists of two segments, one associated with the loading phase and the other with the unloading one. For specific combinations of bit geometry and rock characteristics, a bilinear force/penetration law is representative of the penetration process [2], see Figure 1A. Two questions, however, arise when such a law is to be coupled to a dynamic model of the percussive drilling process.

1. How can the bilinear model be adapted from the frame of single to that of repeated impact loading?
2. How does the timescale associated with the penetration compare to the other ones of the process?

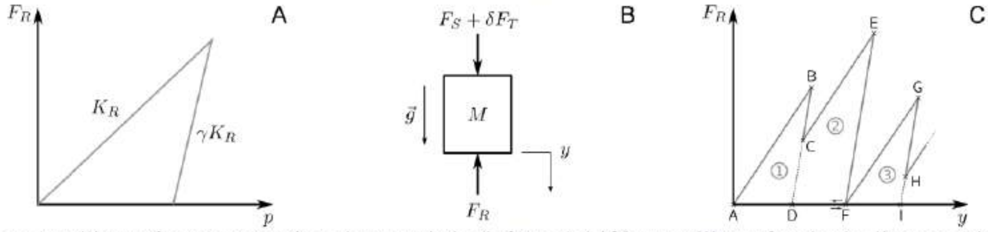

Figure 1: Bilinear force/penetration characteristic (left) and drifting oscillator free body diagram (right).

In an attempt to answer these questions, we study the coupling of a drifting impact oscillator (bit) subject to a periodic impulsive force (percussive activation) as well as a vertical dead load (weight-on-bit), and in contact with an interface whose force/penetration characteristic is based on the bilinear law (bit/rock interaction). Figure 1B illustrates the free body diagram of this one degree-of-freedom oscillator. Central to our analysis are (i) the definition of a relation between the penetration while drilling and the position of the bit and (ii) the identification of the timescales in the problem.

These considerations result in defining the penetration while drilling as the advance of the bit with respect to the final position of the bit/rock interface during the previous drilling cycle plus a residual penetration, has the cycle not completed. Figure 1C illustrates this definition that ensures the continuity of the force on bit. Drilling cycles begin at points A, C, F and H; with A and F following complete cycles, and C and H originating from incomplete ones due to percussive activation. The residual penetration associated with the latter points is defined to verify the continuity of the force on bit. Consequent to this definition is the absence of equilibrium point of the drifting oscillator, for the interaction law is unable to sustain a constant force at zero velocity, e.g. points of peak penetration B, E and G. We therefore complement the bilinear law with a small energy barrier to prevent the bit to enter a new drilling cycle should its kinetic energy be relatively too low, hence avoiding unbounded penetration under the sole action of the static load.

Our analysis also underscores the existence of three characteristic times: (T_1) the activation period, (T_2) the duration of the drilling cycle and (T_3) the duration of the impulsive loading. In the sequel of the analysis presented in [3], we numerically investigate two cases of scale separation in the frame of rigid body motion: $T_1, T_2 >> T_3$ and $T_1 >> T_2, T_3$. In particular, we assess the influence of the model parameters on the average rate of penetration of the bit and qualitatively compare it to experimental results that evidence the existence of an optimal drilling configuration at which penetration is maximized.

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PROBING THE REALISM OF NONSMOOTH DYNAMICS

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ABSTRACT

How may of the processes happening around you involve discontinuities? Stick-slip and impact when objects collide, electrical switches in power systems or in the animal nervous system, changes in behaviour when decisions are made. You'd hope that mathematics could cope with empirical models that involve such discontinuities, and half a century ago Filippov and Utkin [3, 8] wrote the works that would propel the field of nonsmooth dynamical systems into being. They showed not only how to continute dynamics across a discontinuity, but how to understand trajectories that stick to a discontinuity: for rigid bodies in motion this simply models the way sticking occurs due to friction, in electronics this is used for sliding mode control. There are now computational tools for numerical simulation and bifurcation analysis, a fast growing bifurcation theory, and innumerable examples of industrial applications of nonsmooth dynamics (e.g. [2, 7, 5]).

But as the theory of nonsmooth dynamics edges towards maturity, troubling new problems arise: the fundamental breakdown of determinism at "two-fold" singularities, the appearance of unpredictable 'explosions' and sudden spiking, a non-deterministic but non-stochastic form of chaos, and a curse of dimensionality [6, 4]. There were even hints of such phenomena going back half a century [1]. Attempts to resolve such phenomena by reasoning away discontinuities as over-idealized models of something more physical meet with a strange irony that accounting for unmodeled aspects of a system actually makes discontinuous models more exact. In other words, unknown errors can make a system behave more like a crude model than a more refined one, and nonsmooth systems come into their own, complete with non-determinism, jumps to chaos, and explosions. We are only just beginning to understand why this is, and looking to that age old problem – friction – to forge a stronger relationship between the theory and the reality, with new surprises along the way. I'll introduce some of the currents trends in an area wide open with current problems and new ideas.

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SYNCHRONY IN SYSTEMS WITH FRICTION

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ABSTRACT

Friction force is a reaction in the tangential direction between a pair of contacting surfaces. The friction phenomenon can be treated as a result of various factors, i.e., physical properties of the materials of the frictional surfaces, their geometry and topology, relative velocity, and displacement of the bodies in contact [2]-[4]. In this paper an attempt to describe the phenomenon of friction in the context of synchronization of coupled dynamical sub-systems is demonstrated. It creates an innovative concept. According to this idea, dynamics of arbitrary friction oscillator can be described in the following general form:

$$\frac{d\mathbf{x}}{dt} = g(\mathbf{x}, F_T), \tag{1a}$$

$$\frac{dF_T}{dt} = f(\mathbf{x}), \tag{1b}$$

where x $\in \boldsymbol{R}^n$ is a state vector of the response system and F_T is a friction force defined with differential equation (1b) in which the time-delay can be included. The mapping effect results from the solution of Eq.(1b) assuming in the form:

$$F_T = \Psi(\mathbf{x}), \tag{2}$$

where function $\Psi(\mathbf{x})$ describes functional dependence of friction force on variables of the response system (1a). An existence of functional correletion given by Eq.(2) requires some kind of synchrony between the subsystems (1a) and (1b). The identification of its mechanism or a reason of its lack is objective of our work. Preliminary analysis of this case shows some similarity to the so-called *generalized synchronization* [1], which is characterized by the functional relationship analogous to the Eq. (2), but in a unidirectional coupling configuration (master-slave). However, a developing this idea may help to explain the case under consideration. Key point for the detection of synchronization will be determination of the Lyapunov exponents spectrum for the entire system (1a-b), i.e., taking into account the differential model of friction.

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AN ACCELERATION-DEPENDENT FRICTION MODEL SUITABLE FOR FRICTION-INDUCED VIBRATION STUDIES

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ABSTRACT

Experiments on friction induced vibrations on a simple mass on a moving belt system conducted at IIT-Kanpur [1] showed a prominent overshoot of the mass velocity from the belt velocity resulting in negative relative velocities and no prominent stick phase. Also a considerable hysteresis in the forces obtained during the acceleration and the deceleration phases has been observed. These features can only be captured by dynamic friction models involving either internal variables or models wherein the frictional force depends on the relative acceleration along with the relative velocities. The internal variable models could not account for the large overshoot that have been observed in our experiments and we found that most of the existing acceleration-dependent friction models [2,3,4] were not suitable for the study of friction induced vibrations. The model of Guo *et al.* [5] is suitable for friction induced vibration studies but fails to capture the counter-clockwise loop in the hysteretic region observed in our experiments. In this work, we develop a new acceleration-dependent friction model which incorporates all features of the observed experimental data. As part of this work, a new computational scheme is also developed since the discontinuous non-linear governing equations involving acceleration-dependent models are implicit in nature. The effect of the parameters of the new model on the friction characteristics are discussed and these parameters are appropriately chosen in order to obtain a better match between the friction characteristics obtained through numerical simulation with the experimental results. Bifurcation studies are performed with the relevant parameter set and the phase plots obtained through numerical simulations show a decent match with the experimental results.

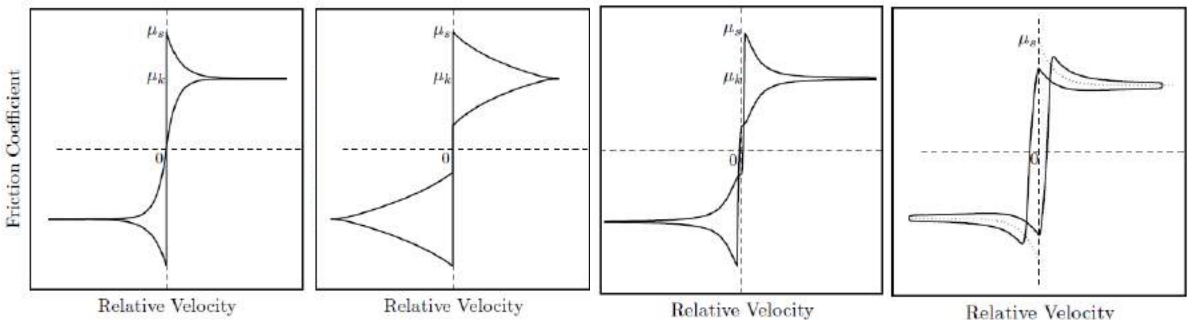


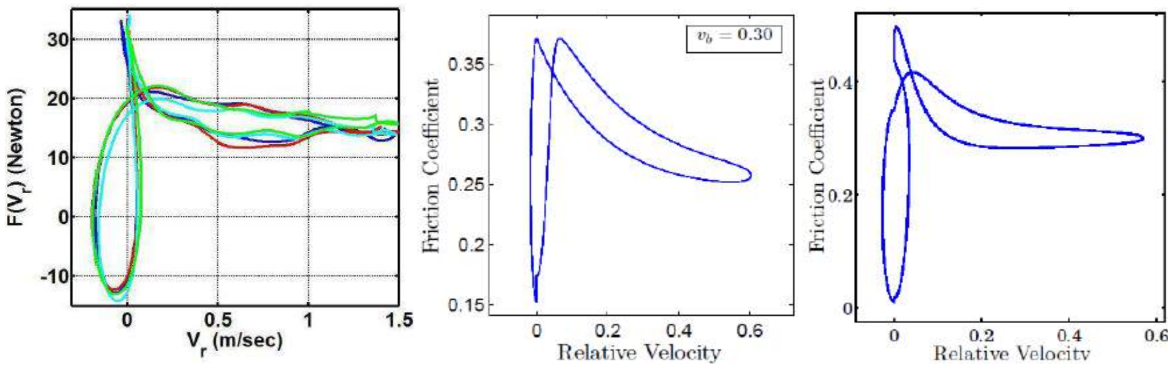
Figure 1: Friction characteristics of some common acceleration dependent friction models (a) model proposed in [2], (b) model proposed in [3], (c) model proposed in [4], (d) model proposed in [5].

The simplest acceleration-dependent friction model [2] involves a friction force whose definition depends only on the sign of the relative acceleration and a typical characteristic associated with it is depicted in Fig. 1a. The models proposed in [3,4] involve dependence of the friction force on the magnitude of the relative acceleration. There is some hysteresis in the pre-sliding regime (around zero relative velocity) in [4] which is absent in [3] as evident in Figs. 1c and 1b, respectively. It can also be observed that the acceleration-dependence in these models [2,3,4] is in essence superposed on the Coulomb model, which results in the loss of the drooping characteristic during relative deceleration. Hence, it cannot explain friction-induced vibrations wherein vibrations are absent at high belt speeds and appear as the belt speed is reduced. The friction characteristics of the model proposed by Guo et al. [5] is shown in Figure 1d. This model incorporates an acceleration-dependent friction term superposed on a velocity-dependent term capturing the drooping characteristics and a Bouc-Wen term with an internal variable to capture hysteresis about the zero relative velocity.

We note that the model proposed by Guo *et al.* [5] can account for most of the experimentally observed characteristics from our experiments. However, the friction characteristics for the acceleration and the deceleration phase in our experiments intersect twice (Fig. 2a) as opposed to only once for the model proposed in [5] (Fig. 2a) in the gross-sliding regime. Furthermore our experiments show an almost constant friction force in the slip phase which is not captured in model proposed in [5]. The experimental behaviour can be captured very well by appropriately modifying the acceleration-dependent term in the model. The modified model for the friction force is

$$
\begin{aligned}
&f = f_d \cdot \xi_e, \\
&\dot{\xi}_e = \rho \left(1 - \left(\sigma \mathrm{sgn}(v)\, \mathrm{sgn}(\xi_e) + 1 - \sigma\right) |\xi_e|^n\right) v, \\
&f_d = \begin{cases} \mu_k + \Delta\mu\, exp(-\tau_1(v)) \\ \qquad + \frac{\mu_a \left(\exp(1-\tau_3(v)) - 1\right)}{1+\tau_1(v)} (1 - \exp(-\tau_2(a)))\, \mathrm{sgn}(av), & \xi_e \cdot \mathrm{sgn}(v) \in [0\ \ 1] \\ \mu_s - \mu_a\,(e-1)\,(1 - \exp(-\tau_2(a_0))), & \xi_e \cdot \mathrm{sgn}(v) \in [-1\ \ 0] \end{cases}
\end{aligned}
\tag{1}
$$

where $\tau_1(v) = |v / v_s|$, $\tau_2(a) = |a / a_s|$, $\tau_3(v) = |v / v_{s_1}|$, and a_0 is the value of the acceleration when the relative velocity becomes zero. v_{s_1} is the relative velocity at which the acceleration and the deceleration branch cut each other. The exponent term involving relative velocity in the numerator of the acceleration-dependent term has been introduced to predict the results as observed in the experiments. A typical friction characteristics predicted by this model for a particular set of parameter values is shown in Fig. 2c and it is very similar to the experimental results shown in Fig. 2a. A detailed study of the effect of the various parameters in the above model on the friction characteristics during friction induced vibrations has been performed and this study has enabled the choice of parameters corresponding to Fig. 2c.



(a) (b) (c)

Figure 2: (a) Experimentally obtained friction characteristics (friction force vs. relative velocity) during friction induced vibrations. (b) Friction characteristics from the model proposed in [5]. (c) Friction characteristics from the new model.

Next, some further analysis on the appearance of friction induced vibrations in a single degree of freedom (SDOF) spring-mass-damper system in frictional contact with a moving belt with the new model is performed. Linear stability analysis is performed and it has helped in identifying the belt velocity corresponding to the loss of stability of the steady solution resulting in the appearance of the vibrations. A detailed numerical bifurcation study is also performed which shows that the bifurcation is subcritical in nature and high-amplitude vibrations appear even in the linearly stable range of belt velocities. The bifurcation analysis also reveals a plethora of complicated response even in the absence of external forcing which can possibly explain the modulations observed in the experiments. More details can be obtained in [6].

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NONLINEAR DYNAMICS OF AN IRRATIONAL OSCILLATOR

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ABSTRACT

Irrational dynamical systems are often occurred in both science and engineeringto describe geometrical nonlinear problems with large deformation [1,2]. In this paper, we consider the nonlinear dynamics of the recently proposed [3] irrational oscillator for dipteran flight mechanism [4] which comprises a lumped mass m, a pair of rigid bars of length l and a pair of springs of stiffness k and length L. Even the springs are linear; this system is strongly irrational nonlinear due to the geometrical configuration.

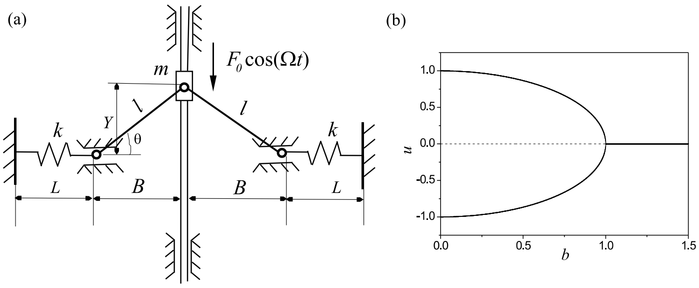

Figure 1: (a) Mass-bar-spring system of depteran flight mechanism (b) equilibrium bifurcation diagram of system.

The dimensionless governing equation of this system can be derived by using Lagrange equation and letting $y = Y/l, b = B/l$ and $\omega_0^2 = 2k/m$, written as

$$\ddot{y} - \omega_0^2 y\left(1 - \frac{b}{\sqrt{1-y^2}}\right) = 0, b > 0, |y| < 1\text{=0}, \tag{1}$$

where Y is the instantaneous vertical displacement of the mass and B is the half length between the ends of the springs at their equilibrium positions with $B \geq 0$. The equilibria of system (1) can be obtained by introducing new variable $u = \dot{y}$, written as

$$(y_0, u_0) = (0,0), (y_{1,2}, u_{1,2}) = \left(\pm\sqrt{(1-b^2)}, 0\right), b > 0, \tag{3}$$

which are shown in Fig.1(b) with the stable and the unstable branches plotted in solid and dashed, respectively. Again the perturbed equation of the dimensionless form with an external harmonic excitation and a viscous damping can also be obtained by letting $\tau=\omega_0 t$, $f_0=F_0/2mk$, $\xi=\delta/2m\omega_0$, and $\omega = \Omega/\omega_0$, written as

$$\ddot{y} + 2\xi\dot{y} - \omega_0^2 y\left(1 - \frac{b}{\sqrt{1-y^2}}\right) = f_0 \cos\omega\tau, b > 0, |y| < 1. \tag{4}$$

Dynamical analysis show the complex nonlinear dynamics of the particular system with a strongly nonlinearity to reflect the natural behaviour of the dipteran flight mechanism of bounded dynamics by avoiding any truncation to the original system. Abundant nonlinear phenomena can be demonstrated by using numerical simulation to the original system including low frequency respond of bursting, the coexisted attractors of different flight modes and also the Wada fractal boundary properties [5].

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BIFURCATION STUDY OF A VIBRO-IMPACT OSCILLATOR WITH DRIFT

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ABSTRACT

Mechanical systems with impacting components are very popular due to their wide engineering applications, e.g. percussive drilling, ground moling and ultrasonic machining. All these applications follow a basic principle: to generate a periodic impact of a machine element upon a certain medium with the purpose of producing deformation or damage to the medium. The efficiency and effectiveness of this process strongly depends on a suitable choice of the control parameters, which, in general, are those driving the external forcing.

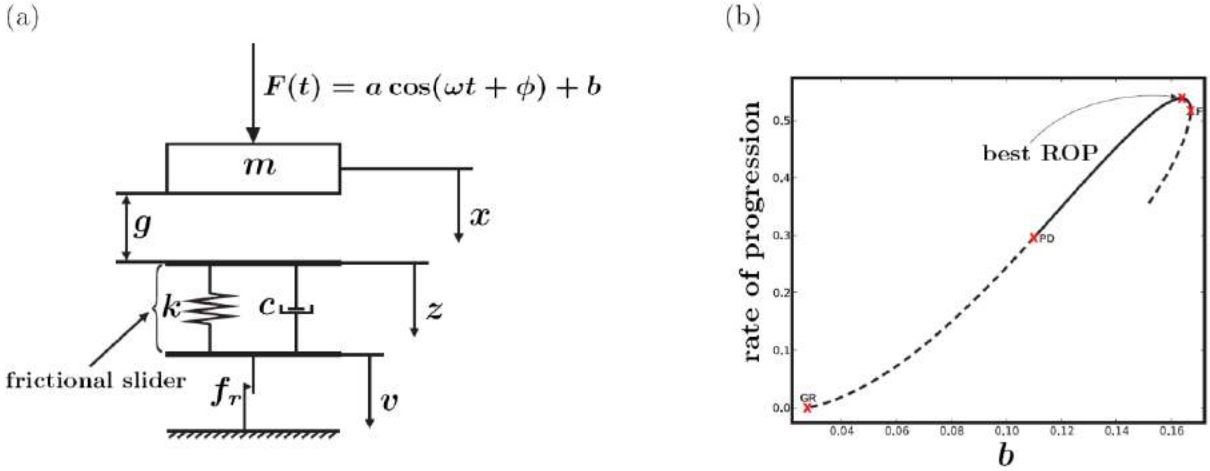

Figure 1: (a) Physical model of the impact oscillator. (b) Behaviour of the ROP as the static force varies.

In Fig. 1(a), we present a physical model describing the interaction between a harmonically forced impactor of mass m that collides with a medium modelled as a viscoelastic slider, as proposed in [1]. Systems of this type are often modelled by piecewise smooth ODEs, due to the presence of non-smooth or discontinuous phenomena in the physical model. This results in fascinating and complex dynamical behaviour, ranging from various periodic responses to chaotic motion [1], [2]. The present work extends the previous investigations undertaken by the CADR within Aberdeen University to gain an insight into the bifurcation structure of the underlying system. Specifically, the system is analyzed numerically by means of the software TC-HAT [3], a toolbox of AUTO 97 allowing numerical continuation and bifurcation detection of periodic orbits in non-smooth dynamical systems. In particular, this study will allow us to achieve a deeper understanding of the role played by each of the control parameters in the system response and, in particular, how they affect the rate of progression (ROP), which is a measure of the efficiency of the system. An example of the possible behavior of the ROP with respect to the control parameters can be observed in Fig. 1(b). In this picture, we can identify fold (F), period-doubling (PD) and grazing (GR) bifurcations of limit cycles, as well as the point at which the ROP achieves a maximum value.

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CUSP OF A CONE OPENING UPWARD SEEMED TO BE AN ATTRACTING LAUNCHER UNDER HORIZONTAL PERIODIC FORCING: OCCURRENCE OF INTERMITTENTLY EXPLOSIVE CHAOTIC MOTIONS

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ABSTRACT

Previous theoretical and simulation investigations [1] have illuminated the relationship between nonlinear characteristics and outcomes (steady states) arising from nonlinear systems, that is, the basic behavior of a mass point in a rotating external force around the vertical axis, which is constrained on a surface of revolution opening upward around the same vertical axis. We observed jump phenomena in the amplitude or the altitude of the horizontal circular motion under variations of the angular velocity of the external force almost-stationary, and obtained the dependence of the of the jump phenomena on the curvature of the surface of revolution, i.e., discovered an interesting qualitative difference, which is separated by parabolic surface of revolution.

This study deals with the behavior of a mass point in a rotating external force around the vertical axis, which is constrained on a cone opening upward around the same axis. Let us take the vertical *z*-axis upward, and generate a curve of the constrained surface by $z = x$, i.e. cone $z = \sqrt{x^2 + y^2}$. Of course, the cusp is an origin and the (*x*, *y*) - plane is horizontal. A motion of a constrained mass point is described by the following normalized equation: (1) where (2). The approximate solution of Eq. (1) is described by: (3) where the amplitudes $a(t)$, $b(t)$ and $c(t)$ are slowly varying functions of time *t* as compared with externally-forcing signal cos(ω*t*), and sin(ω*t*). Then the well-known normal means leads to the autonomous equation: (4) where (5).

The frequency response curves and phase portraits are similar to Fig. 3 and Fig. 5 shown in Ref. [1], except for non-resonant states. The non-resonant state of Eq. (4) is given by the origin, i.e., the cusp of cone, which seems non-simple genuine singular point. Simulation experiment for autonomous equation (4) shows that cusp has asymptotical stability, i.e., non-logarithmic spiral, but for non-autonomous original Eq. (1) cusp seems to be launcher for a mass point. This matter is inferred by inspecting the third equation of Eq. (1), because of the relation $r = z$ is formed for cone.

To have an overview of the steady motion observed in this system, we may outline for it as follows: Either resonant or chaotic motions occur depending on the control parameter values F and ω. Distinctive behavior of chaotic motions may be seen in its intermittency of shooting interval and attractive focus, sketch of this situation is similar to the Fig. 9 in Ref. [2] with the caption "Shilnikov's spiral chaos." It is to be noted that this behavior is observed on a cone under periodic excitation, described by Eq. (1), i.e., motion takes place in four dimensional space (*x*, *u*, *y*, *v*).

$$\left.\begin{aligned} \frac{dx}{dt} &= u \quad & \frac{du}{dt} &= -ku + Fcos(\omega t) - \frac{x}{r}\Lambda(t) \\ \frac{dy}{dt} &= v \quad & \frac{dv}{dt} &= -kv + Fsin(\omega t) - \frac{y}{r}\Lambda(t) \\ \frac{dz}{dt} &= w \quad & \frac{dw}{dt} &= -kw - \frac{g}{l} + \Lambda(t) \end{aligned}\right\} (1)$$

$$\left.\begin{aligned} r^2 &= x^2 + y^2 \\ \Lambda(t) &= \frac{1}{2}\Big[\frac{g}{l} + \frac{1}{r}(xFcos(\omega t) + yFsin(\omega t) + u^2 + v^2) \\ &\quad - \frac{1}{r^3}(xu + yv)^2\Big] \end{aligned}\right\} (2)$$

$$\left.\begin{aligned} x(t) &= a(t)\cos(\omega t) - b(t)\sin(\omega t) \\ y(t) &= b(t)\cos(\omega t) + a(t)\sin(\omega t) \\ z(t) &= c(t) \end{aligned}\right\} (3)$$

$$\left.\begin{aligned} \frac{da}{d\tau} &= \frac{1}{2\omega^2}\left[-k\omega a + \left(\omega^2 - \frac{g}{l}\frac{1}{r}\right)b\right] \\ \frac{db}{d\tau} &= \frac{1}{2\omega^2}\left[-\left(\omega^2 - \frac{g}{l}\frac{1}{r}\right)a - k\omega b - F\right] \\ 0 &= -\frac{g}{l} + \Lambda \end{aligned}\right\} (4)$$

$$\tau = \omega t, \;\; r^2 = a^2 + b^2 \qquad (5)$$

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FINITE ELEMENT MODELLING OF ROTARY IMPACT DRILLING

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ABSTRACT

Low dimensional analytical models [1,2] have been introduced to study the dynamic responses of the Resonance Enhanced Drilling (RED), a new drilling technology developed at the University of Aberdeen [3]. However, while these studies provide insight into the dynamic behaviour, they do not consider the influence of the drill-bit geometry and distribution of cutters on the contact forces and rock fragmentation. In current work, a comprehensive numerical analysis of the drilling and rock fragmentation processes is carried out using nonlinear finite elements method (FEM) to provide a deeper understanding of the dynamics and the stress field associated with a steadily propagating fracture.

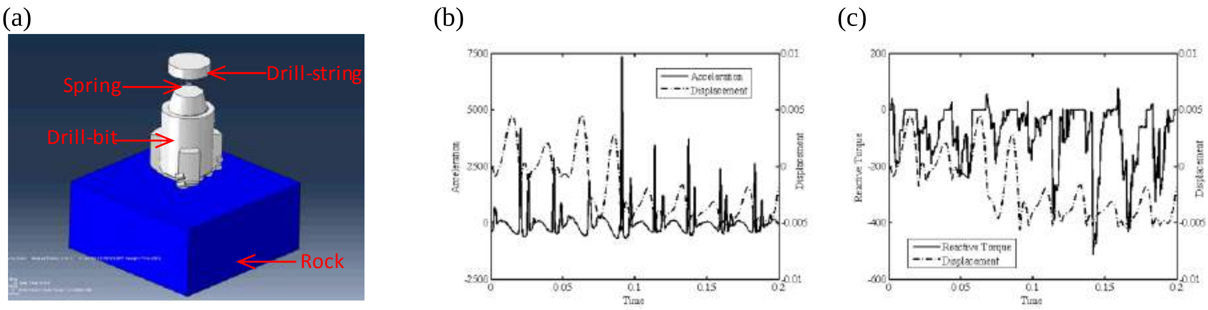

Figure 1: (a) A solid model for rotary impact drilling, (b) time histories for acceleration and displacement, and (c) time histories of reactive moment and displacement.

Figure 1(a) shows schematically a model representing the RED system, which is a rotary impact drilling. It consists of a typical drill-bit attached to a drill-string modelled as a spring, and a block representing a rock formation being drilled. Noting that the finding of Detournay and Defourny [4] explaining that a rock cutting process involves a combination of brittle and plastic fracture, the Johnson-Cook damage model and the linear modified Drucker-Prager plasticity models [5] were used here to describe the constitutive law and fragmentation of the rock. The base of the model rock was fixed, a rotary speed and harmonic force was applied to the drill-bit, while a constant load was applied to the top mass in order to induce impacts. In addition, appropriate contact constraints were applied between the drill-bit and the rock block, and between elements to ensure rock fragmentation is captured.

The simulation was carried in ABAQUS Explicit Dynamics software. The resulting dynamic response of the drill-bit are shown in Fig. 1(b) for the vertical displacement and acceleration, while in Fig 1(c) the corresponding reactive torque is depicted as a time history. It is noted that sudden changes in the acceleration occurred simultaneously as abrupt reduction in rotational velocity when impacts occurred. It is also observed that the borehole developed after the damaged elements were removed forming a circular cylinder with the geometry of the drill-bit. The reactive torque dropped at zero value whenever the drill-bit lost contacct with the drilled formation. These numerical results are consistent with observation from experiments on a prototype RED module. The presented work confirms the viability of the application of FEM to drilling processes and also suggests a possibility of its wider use in the optimization of drill-bit design.

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COMPUTATIONAL METHODS FOR NON-SMOOTH DYNAMICAL SYSTEMS

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ABSTRACT

Nowadays there are available computational tools for the study of qualitative changes of linear and nonlinear dynamical systems but these tools are mainly applicable to smooth systems. However, there are many engineering applications that are non-smooth, namely systems with motion dependent discontinuities such as impacts, friction, clearances, or a combination of them.

Nonlinear dynamics has developed an array of mathematical techniques to study numerically stability, which can be local, global or structural. Specifically, the most popular include bifurcation diagrams, Poincaré maps, and basins of attraction. The latter will be the object of the current studies, where we focus on stability, co-existing attractors, creation or destruction of equilibriums as one the system parameters is varied.

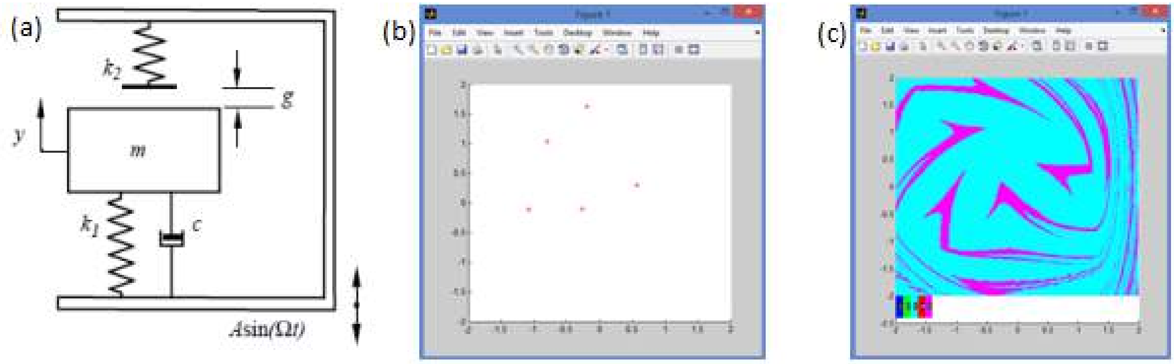

Figure 1: Physical model of the analysed bilinear oscillator adopted from [2] in (a), its Poincaré section showing a Period-5 orbit in (b), and its basins of attraction constructed after fixing the parameter values of the system.

Thus, the present work makes use of techniques mentioned above by using two different approaches with respect to non-smoothness of the systems, proposed in [1], and describes the computational methods involved such as bisection, interpolation, and the use of the sigmoid function applied to a piecewise linear oscillator. The first approach is based on dividing the global phase space in local subspaces where the system is smooth, detecting the discontinuities and the precise time when they occur, and gluing the local solutions in order to obtain the global one. The second approach is based on replacing the non-smooth functions of the system by smooth ones.

Finally, the algorithms considering these approaches are programmed by using the software Matlab and used to elaborate Poincaré maps, basins of attraction and bifurcation diagrams for the non-dimensional mathematical model of the bilinear oscillator shown in part (a) of the Figure 1 and proposed in [2] in order to look for the system parameters where the exact and smoothed approach produce very different basins of attraction.

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MODAL DAMPING PREDICTION FOR VIBRATING SOLIDS: CONSTITUTIVE MODELS AND FINITE ELEMENT COMPUTATIONS

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ABSTRACT

First principles prediction of the internal vibration damping of engineering components is not routine in finite element (FE) packages. With such predictions, designers would be able to assess the noise and vibration levels in engineering systems right from the component design stage. To this end, we revisit the modelling of internal material dissipation under spatially variable triaxial stresses, a research topic that peaked five decades ago.

Internal energy dissipation in many materials, per stress cycle and per unit volume, is found to be frequency-independent and proportional to some power ($m \geq 2$) of a suitable equivalent stress amplitude ($D = J\sigma_{eq}^{m}$) [1-4]. Definition of this equivalent stress amplitude under arbitrary triaxial stress states remains an open question. Such a definition is needed for computing modal damping of arbitrary solid bodies using finite element (FE) packages. In this paper we model the macroscopic dissipation due to numerous randomly dispersed, microscopic dissipation sites; the resulting dissipation model is used in FE prediction of modal damping in solid objects.

We have two simple choices. Constrained by empirical evidence, we consider rate-independent micromechanics only. Two dissipative phenomena that can be modeled as rate independent are (i) Coulomb friction, and (ii) ambient-temperature plasticity with its multitude of microscopic phenomena including dislocation movements. Both possibilities involve shear-driven dissipation. There are two corresponding simple mathematical models. The first is a flat crack in an elastic material, with Coulomb friction between the crack faces [5], wherein there is a first order coupling between the normal and shear stress components on the crack face. The second model, motivated loosely by dislocations, considers zero order coupling between normal and shear stresses, i.e., slip occurs when the shear traction reaches a predetermined value, independent of the normal stress.

For the first model, the macroscopic dissipation is obtained by Monte Carlo averaging of the dissipation from many randomly oriented non-interacting microcracks, and finally fitted using a multivariate polynomial. We suggest that such a model might be appropriate for ceramics rather than metals. This model, in the absence of prestress, always gives $m = 2$. For the second model, we assume a random distribution of flaw strengths (Weibull-distributed) and random orientations, and make easier analytical progress than in the first case. In particular, we can incorporate arbitrary $m > 2$. When m is between 2 and 6, the net dissipation is accurately described by a power of the distortional strain energy (and exactly so, for $m = 2$ and 4). For larger m, separate asymptotic formulas are found, showing that the dissipation is not a function of distortional strain energy alone.

We finally demonstrate use of the above dissipation models to compute the modal damping ratios of an arbitrary solid object (see, e.g., Fig. 1). Our finite element calculation of the effective damping ratio ζ_{eff} uses modal analysis results from ANSYS complemented by our own volume integrals for the dissipation. We close with an elementary discussion of some issues in shape optimization for damping in engineering components.

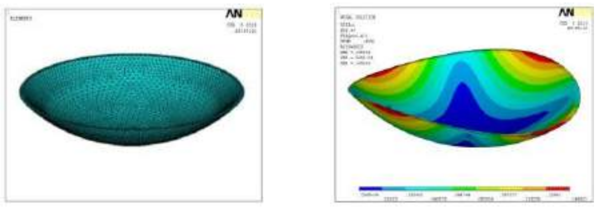

Figure 1: Part of a spherical shell ($r = 1$ m, $\theta = 0$ to $\pi/4$, $\phi = 0$ to 2π; $t = 0.01$ m). 10-noded tetrahedral elements were used. ζ_{eff} for the 1st mode is 1.132 $JE/2\pi$. Here J is a material constant, E is the Young's Modulus of the material, and the 1.132 is found using FE computations. The coefficient will change with the shape of the body, and for different modes of the same body.

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SPIKE TRAIN PATTERN IN A NETWORK MODEL OF THE OLFACTORY BULB

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ABSTRACT

Information about the environment is generally encoded into spike sequences by neurons in animal sensory nervous systems [1]. Many different coding mechanisms are usually discussed in the literature, including spiking rates, time coincidence, time ranks, spatial pattern, and spatiotemporal patterns. The main olfactory bulb receives signals from the population of olfactory receptor neurons (ORN) and transmits signals to the olfactory cortex and other brain regions. Mapping of the sensory inputs has revealed that each odorant produces a reproducible spatial pattern of activation in the glomerular layer of the bulb. In the olfactory bulb, the temporal structure of neuronal activity appears to be important for processing odor information; especially odour-evoked synchronization is functionally relevant for olfactory discrimination.

We used a detailed, realistic model of the mitral-granule cell circuits in the olfactory bulb that introduced by Bhalla-Bower [2] to investigate the spatio-temporal processing of odor information. A single compartment that includes voltage-dependent currents described by Hodgkin-Huxley kinetics was used to model mitral cells. Their membrane potentials were calculated with the equations:

$$C_M \frac{dV}{dt} = -\frac{1}{R_m}(V - E_l) - I_{Na} - I_{Kfast} - I_{Ka} - I_{Ks} - I_{Nap} - I_s - I_e \, , \qquad (1)$$

where V is the membrane potential, C_M is the membrane capacitance, R_m is the input membrane resistance, and E_l is the leak reversal potential. I_i are ionic currents (i=$Na,Kfast,Ka,Ks,Nap$). I_S and I_e are synaptic and external currents respectively. The mitral cells have two sodium currents I_{Na}, I_{Nap} and three potassium currents: I_{Kfast}, I_{Ka} and I_{Ks}. All these currents are described by the equation:

$$I_i = g_i m^M h^H (V - E_l) \qquad (2)$$

where g_i is the maximal conductance and E_i the reversal potential. The activation and inactivation variable m and h raised to the power M and H respectively follow the kinetic equation:

$$\frac{dm}{dt} = (m_\infty - m)/\tau_m \qquad (3)$$

$$\frac{dh}{dt} = (h_\infty - h)/\tau_h \qquad (4)$$

All equations and parameters for the mitral cells and granule cells are taken form [3]. Based on the olfactory model, our analysis shows that maximal conductance will have obviously effects on the spike train pattern of single neuron: maximal conductance of *Na* decide the top value of the membrane potential, however, the frequencies of firing can be controlled by the maximal conductance of *Nap* and *Ks*. We also investigated the effect of odor intensity on both the spatial activity pattern and the synchronization of the network by stimulating the model with the same odor input at different intensities. The degree of synchronization increases as the odor intensity is increased.

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BIFURCATION ANALYSIS IN N-SCROLL MODIFIED CHUA'S CIRCUIT

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ABSTRACT

The bifurcation theory is very important for understanding the dynamical behavior of dynamical systems. The bifurcations analysis in smooth vector fields are well understood [1], and the attentions for non-smooth vector fields have been attracted in recent years [2].

Chua's circuit has become a paradigm for complex oscillatory dynamics and chaos arising in simple electronic nonlinear circuits . A modified Chua's circuit is proposed in 2001 by Tang et al. as follows [3]:

$$\begin{cases} \dot{x} = \alpha\left[y - f(x)\right] \\ \dot{y} = x - y + z \\ \dot{z} = -\beta y \end{cases} \tag{1}$$

where $f(x) = \begin{cases} \dfrac{b\pi}{2a}(x - 2ac), & x \geq 2ac \\ -b\sin(\dfrac{\pi x}{2a} + d), & -2ac < x < 2ac \\ \dfrac{b\pi}{2a}(x + 2ac), & x \leq -2ac \end{cases}$,

with the parameters $c = n - 1$(n integer), $\alpha = 7, \beta = 15, a = \dfrac{\pi}{2}$ and b. The system (1) is first order differentiable with $f(x)$ as a piecewise function. It can generate an n-double scroll chaotic attractor for proper parameters and play an interesting role in chaos control and synchronization [4-5].

Here we depict some bifurcations of equilibrium points of this system. It is shown that there exist $2c+1$ (that is, $2n-1$) equilibrium points given by $(\pm 2ak,\ \ 0,\ \ \mp 2ak)$ $(k = 0, \pm 1, \cdots, \pm c)$ in the n-scroll modified Chua's circuit (1). After their stability conditions are obtained, we found $n-1$ or $n-2$ Hopf bifurcations may coexist if the parameters are properly chosen, which means that $n-1$ or $n-2$ periodic solutions may appear simultaneously. Hence, we can obtain the desirable number of periodic solutions in this system by controlling the parameter n. Furthermore, we indicate that the previous Hopf bifurcation conditions of equilibrium points where the systems are differentiable sufficiently many times (smooth) are invalid for equilibrium points where the systems are first order differentiable. The theoretical results are verified by numerical simulations.

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DYNAMICS OF DRILL-STRINGS WITH COSSERAT ROD THEORY

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ABSTRACT

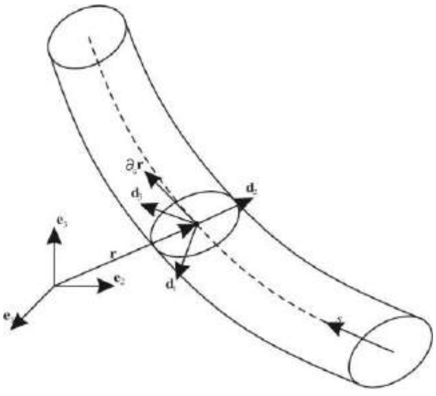

Figure 1: Cosserat rod configuration

A drill-string presents a highly complex dynamical behaviour, which can result from presence at the same time of different phenomena such as torsional, lateral and axial vibrations, stick-slip, bit-bounce and others. The occurrence of these phenomena, especially stick-slip vibrations, lead to reduction of depth of penetration and may cause failures and wear to the drilling equipment, thus increasing considerably drilling time and its costs. Therefore it is of utmost importance to understand them, since in most cases they are detrimental to the drilling process. We follow the idea of Tucker and Wang [1] to use Cosserat theory to describe the drill-string, which due to its length, reaching several km, is a slender structure. The motion of Cosserat rod is described in the inertial Cartesian frame of reference (x, y, z) with right-handed orthogonal basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$. It is assumed that plane cross-sections subjected to deformation, undergo only rigid rotation. Therefore these cross-sections preserve their shape and area, remaining plane after the deformation, what comes from the Bernoulli hypothesis. The Cosserat rod is described by its neutral axis $\mathbf{r}(s,t)$, known also as Cosserat curve and 3 orthogonal unit vectors $\boldsymbol{d_i}(s,t), (i = 1,2,3)$ called Cosserat directors, where s represents length parameter and t time. Directors $\mathbf{d_1}(s,t)$ and $\mathbf{d_2}(s,t)$ lie in the plane of rotated cross-section in contrast to director $\mathbf{d_3}(s,t)$, that is perpendicular to it. This configuration is shown in Fig. 1. The cross-section orientation of rod's axis $\mathbf{r}(s,t)$, is determined at any moment of time by $\boldsymbol{d_i}(s,t)$ satisfying the condition: $v_3 = \partial_s r \cdot \mathbf{d_3} > 0$, what means that the local ratio of deformed length to reference length of the axis doesn't reduce to 0 and implies that total shear, where plane described by directors $\mathbf{d_1}(s,t)$ and $\mathbf{d_2}(s,t)$ becomes tangent to the curve r(s; t), is not possible to take place. Following Liu *et al*. [2] we build nonlinear finite element model, that takes into account coupling between axial, torsional and transverse vibrations. Finally basing on Lagrange approach we assemble finite element equations of motion, which are written in matrix form, with appropriate mass (12x12) and stiffness (12x12) matrices and nonlinear stiffness vector (12x1). At this stage the model assumes linear viscous damping for every degree of freedom of an element. We validate the model, by comparing theoretical torsional buckling conditions with the behaviour of the assumed model. The theoretical predictions in form of $\frac{M^2}{\left(\frac{2\pi EJ}{l}\right)^2} + \frac{F}{\frac{\pi^2 EJ}{l^2}} = 1$, where M is a couple end torque, F is a pair of opposite axial loads (For $F < 0$ the beam is in tension, and for $F > 0$ in compression), E Young's modulus, J torsional rigidity, l string's length are confirmed for a drill-string in tension, which is its usual working condition. An example of helically buckled drill-string is shown in Fig. 2.

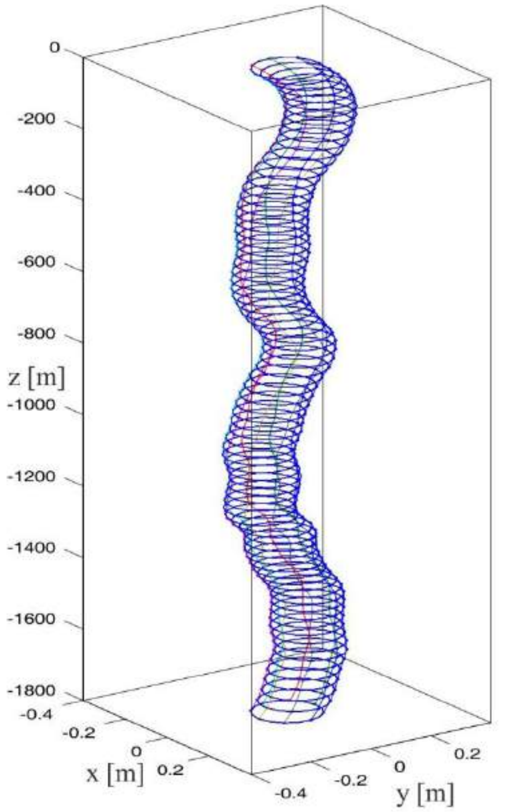

Figure 2: Helically buckled drill-string in tension, F=100 N, M=97600 Nm, L=1800 m, $r1$=0.1 m, $r2$=0.0945 m

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INFLUENCE OF TENSION VARIATION ON VIV: A PRELIMINARY STUDY

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ABSTRACT

Vortex-induced vibration (VIV) of a slender, flexible structure tends to be more complex than the dynamic behaviour of a rigid cylinder subjected to the same flow conditions. This is due mainly to the influence of several parameters, which have an important impact on the stiffness of the flexible system. One such parameter that has received relatively little attention is the variation of the axial force typically applied at the top of a vertically positioned pipe. Variation, which can be intentional (e. g. stiffening of the riser to avoid resonant responses) or can be imposed externally by the environment (e. g. an offshore riser attached to a floating platform following the motion of waves), modifies the stiffness of the structure and hence its resonant response. In order to determine the difference in responses between cylindrical structures with linear and nonlinear stiffnesses, two systems are investigated in this preliminary analysis.

Once reduced to the subspace of a singular normal mode, the equation of motion for the linear system can be written as:

$$\ddot{v} + [\omega^2 + P\mu \sin(\omega_P \tau)]v = -a\,\Omega\,\dot{v} + b\,\Omega^2\,\mathrm{q} \tag{1}$$

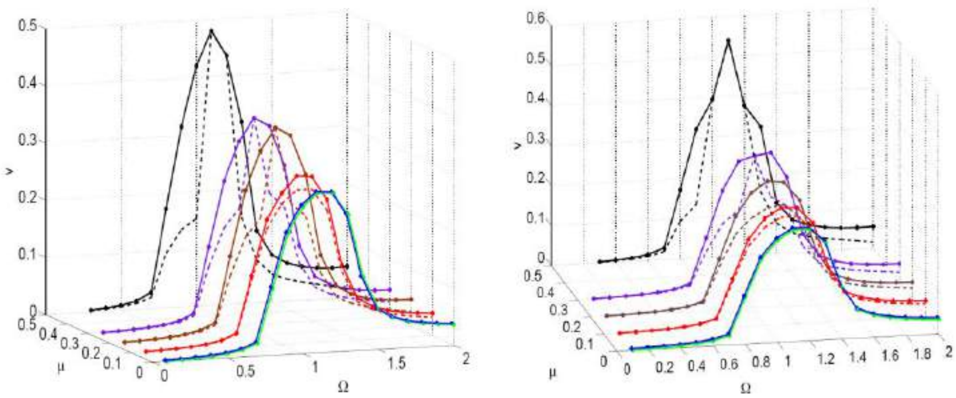
For the system with weakly nonlinear properties the equation of motion is expressed as:

$$\ddot{v} + [\omega^2 + P\mu \sin(\omega_P \tau)]v \; + \; \xi v^3 = -a\,\Omega\,\dot{v} + b\,\Omega^2\,\mathrm{q} \tag{2}$$

Using the experimental findings by Nakano & Rockwell [1], parametric excitation is set to match the resonant frequency of the structure. Excitation due to the interaction with the flow of the surrounding fluid is modelled as an external force governed by the Van der Pol equation in the Facchinetti wake oscillator model [2]:

$$\ddot{q} + \lambda\,\Omega\,[\gamma\, q^2 - 1]\,\dot{q} \; + \; \Omega^2 q = A\,\ddot{v} \tag{3}$$

Comparison of both systems demonstrate that the beating response can be adequately controlled by the strength of the parametric excitation (μ) only over a portion of the lock-in region, where the beating oscillation is suppressed also in the nonlinear structure. Furthermore, it was observed that at lower strengths (μ<1) parametric excitation has little influence on the shape of the resonant region, while significantly larger amplitudes occur when μ is increased.



(a) (b)

Figure 1: Influence of μ on the shape of the response curve for the structural variable *v*: (a) linear structure, (b) nonlinear structure.

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DYNAMICS OF AN ACTIVE AUTOPARMETRIC SYSTEM

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ABSTRACT

Autoparametric systems belong to a special class of nonlinear dynamical systems which induce so-called parametric vibrations. Their special feature is that vibrations are caused by internal coupling of at least two subsystems.

This work focuses on autoparametric vibrations of the system composed of the nonlinear oscillator with the attached pendulum (Fig.1). Theoretical and experimental investigations [1]–[3] have shown that harmonic excited pendulum system may undergo complex dynamics. Here we propose to use a combination of the semi-active damper together with the nonlinear spring. The spring will be made from a shape memory alloy (SMA) and the damping will be realised through a magnetorheological dampers (MRD). The MD damper will be an off shelf device [RD 1092-01], which characteristics have been investigated in [4].

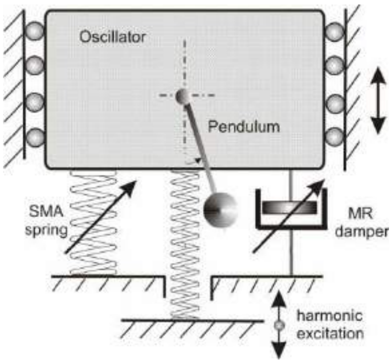

Fig.1 Physical model of an autoparametric system with a MR damper and a SMA spring.

The system will be model numerically and verified experimentally on a custom made experimental rig. Specifically we will investigate the nonlinear resonances and their influence on the complexity of the system dynamics. Both elements, SMA and MRD, are modeled and studied with hysteretic loops. The multinomial model was introduced to describe the hysteretic nonlinear relationship between strain, stress and temperature relationship of SMA. The MR damper is circumscribed by Bingham model, which includes displacement and velocity parts. The ultimate goal of the work is the effective vibration isolation which can be used in various suspension systems.

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AIRCRAFT IMPACT INTO REINFORCED CONCRETE STRUCTURE

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ABSTRACT

Analysing the consequences of potential aircraft impact into engineering structures is an issue of high importance since September 11, 2001. Majority of the theoretical and experimental investigations is based on and follows the model of Brazilian civil engineer Jorge D. Riera [1]. The Riera model consists of a perfectly rigid target structure and a deformable missile (aircraft) that collides in normal direction. The Riera model is widely accepted, its assumptions have been verified by several experiments. However, in certain cases the applicability of this model has not been clarified in details, e.g. reviewing how conservative is the assumption that the target structure is perfectly rigid. Beside the original Riera model (Fig. 1.a) this research investigates a model that is based on the Riera model but takes into consideration the possible deformations of the target structure (Fig. 1.b).

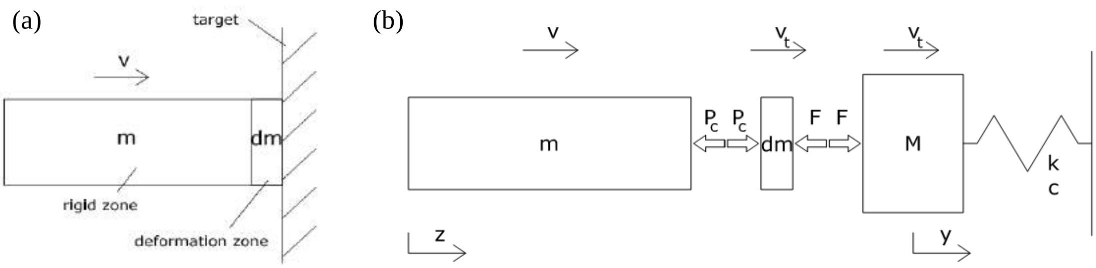

Figure 1: Schematics of the original Riera model (a) and the elastic Riera model (b)

In this elastic Riera model the missile consists of an undamaged part that has the mass of *m* and a mass *dm* that is the instantaneously breaking cross-section of the aircraft. The target structure is modeled by a one-dimensional vibrating system (having the mass *M*, spring constant *k* and damping coefficient *c*). Force P_c is the buckling force that causes the breaking of the missile, while *F* is the reaction force between mass *dm* and the target that causes mass *dm* slowing down from velocity *v* to the velocity of the target (v_t).

Analytic calculations based on the elastic Riera model are compared to the results of an explicit dynamic final element model. The differences of the analytical and numerical results and their potential origins (propagation of shockwaves, important parameters and their harmonization) are also discussed.

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DRILL-BIT FORMATION INTERACTION INVESTIGATED USING AN IMPACT OSCILLATOR

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ABSTRACT

In the process of drilling, the rate of penetration will vary widely due to the heterogeneity and anisotropy of formations, which is one of the main reasons for drilling failure. However, as a consequence of the interaction between drill bit and formations, the axial vibration condition of drill bit will be changed when different types of formations with different stiffness are met, which provides a potential method to check the formation properties. In this presentation, we consider an impact oscillator with a one-sided elastic constraint [1] as a simplified representation of the impact vibration of drill-bit. The effect of varying 'formation' properties was investigated by varying the stiffness of the secondary constraint. For the experimental set-up (see Fig. 1 a), the length of the parallel leaf springs was fixed while the length of beam was adjusted constantly to make a series of varying stiffness ratios. Moreover, both the gap and the excitation amplitude were fixed but the excitation frequency gradually increased in a suitable range.

From the results of both experimental analysis and continuation simulation using the toolbox TC-Hat, the same trend has been observed: as the stiffness ratio is increased, there is an increase in the frequency ratio at which bifurcations occur (see Fig. 1 b). This strong dependence is a useful result, since it demonstrates how changing formation properties will influence the dynamic response of the drill bit in a real system. In the future, this dependence will be used for a real drilling system, to evaluate the formation properties.

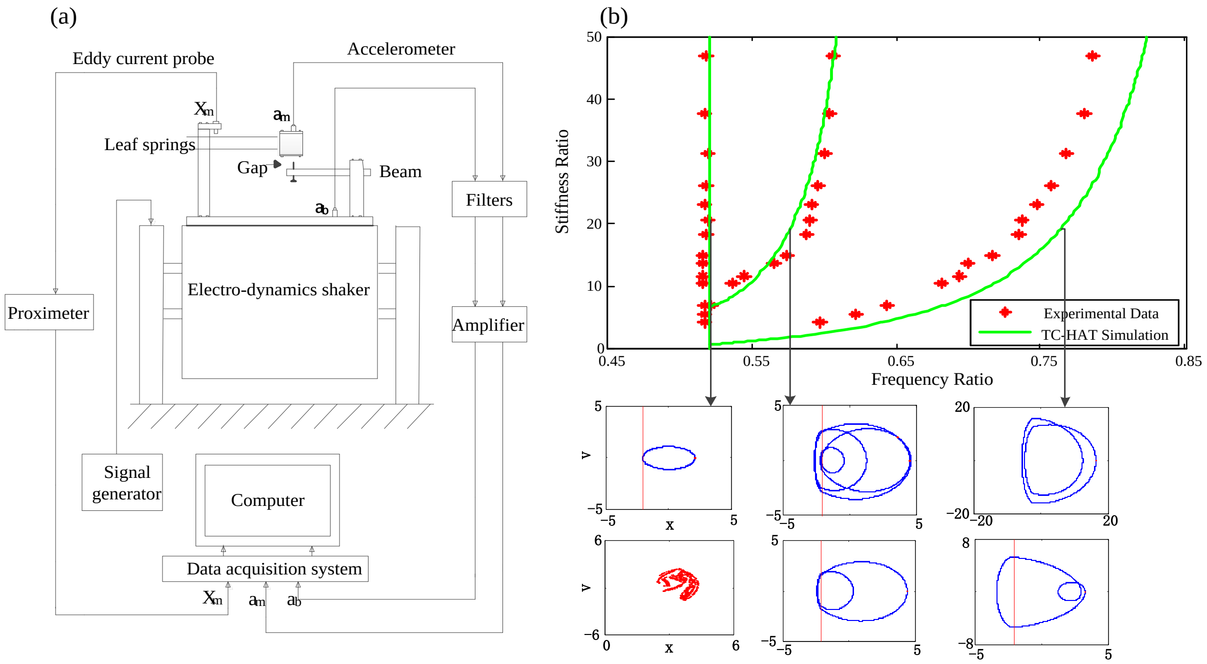

Figure 1: (a) Schematic diagram of impact oscillator experimental set-up. (b) The experimentally obtained grazing, period-doubling bifurcation, and fold bifurcation points (red points) are compared with that obtained by continuation simulation using TC-Hat (green curves). Specifically, from left to right in (b), the first straight line represents grazing; the second curve shows the period-doubling bifurcation; and the third one displays the fold bifurcation. In addition, there are three lists of small windows below, which, respectively, show the before and after conditions about these three kinds of vibration condition changing.

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TORSION PROPERTIES OF PERIODIC STATES

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ABSTRACT

In nonlinear dissipative dynamical systems, parameters sets correspondent to periodic states are immersed in regions of parameters correspondent to chaotic states. These islands of periodicity are called *periodic windows* and their existence has been numerically and experimentally reported in previous works [1-6]. Commonly, in two-dimensional parameter spaces, the periodic windows are aligned in sequences, and the periodic states inside a periodic window are connected with periodic states of other-windows by period-adding rules [7,8]. This fact is very important to technical applications once it provides elements to know the period of states in a certain periodic windows by knowing the period of states of other periodic windows. On the other hand, the periodic states have others intrinsic properties, for instance, the *torsion number*, defined as the number of twists that local flow perform around a given periodic state [9-11]. So, as it was found for the window periods, it is possible to establish addition rules to the torsion number of periodic states (Figure 1). In fact, we propose a torsion-adding rule for periodic states inside periodic windows in a given sequence. Additionally, combining the period and the torsion adding rules, we obtain an expression to the *generalized winding number* whose asymptotic limit provides the winding number of the accumulation region of a periodic windows sequence [12].

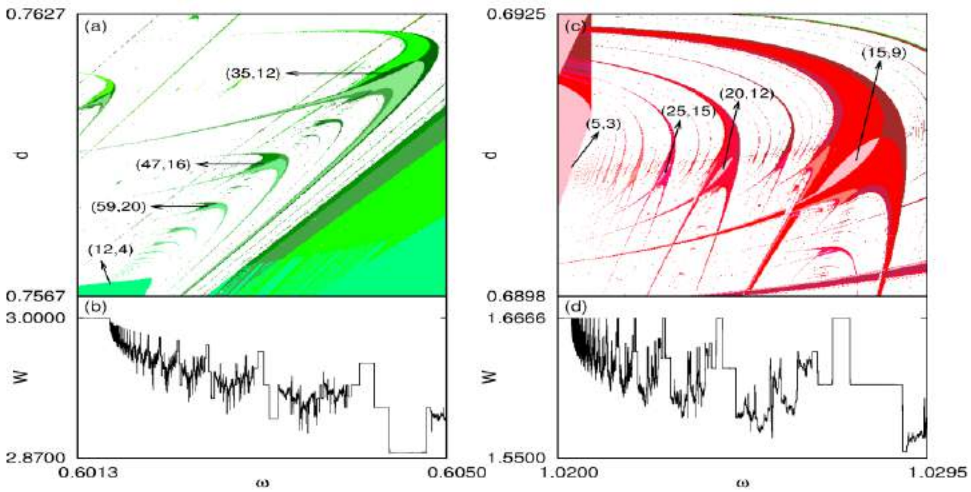

Figure 1: Periodic window sequences in the parameter space of a nonlinear oscillator. The numbers (n,m) are the torsion and the period of the correspondent area inside the window.

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NONLINEAR CONTACT PROBLEMS IN CRACKS' DYNAMICS

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ABSTRACT

The modern engineering standards demand that the components' design incorporates an awareness of various safety factors. Consequently the great attention is paid to the failure analysis as an essential tool to improve materials' reliability, which helps to prevent accidents and disasters caused by the unpredicted fracture. It is common knowledge that all existing structural materials contain various inter- and intra-component defects (in particular – cracks), the presence of those considerably decreases the strength and the reliability of structures.

Under deformation the opposite faces of the existing cracks interact with each other, altering significantly the stress fields near the crack tips. The analysis of static problems demonstrates that the contact interaction considerably changes the solution. It takes on special significance for the case of high rate deformations as found in impact and high-frequency dynamics, which covers an extremely wide range of situations, where the contact interaction can change the response substantially. The complexity of the problem is further compounded by the fact that the contact behaviour is very sensitive to the material properties of the two contacting surfaces and parameters of the external loading. As a consequence, due to the non-linearity of the problem and substantial computational difficulties, researchers almost always neglect effects of the crack closure even for the simplest case of isotropic homogeneous cracked solids. Therefore the real stress-strain distribution is ignored in spite the fact that in some cases the difference between comparable quantities can exceed 50%, e.g., see Fig. 1, where the stress intensity factors, obtained neglecting the crack closure and taking it into account, are given vs. the wave number and the angle of the wave incidence for the oblique harmonic loading of a penny-shaped crack located in a homogeneous material.

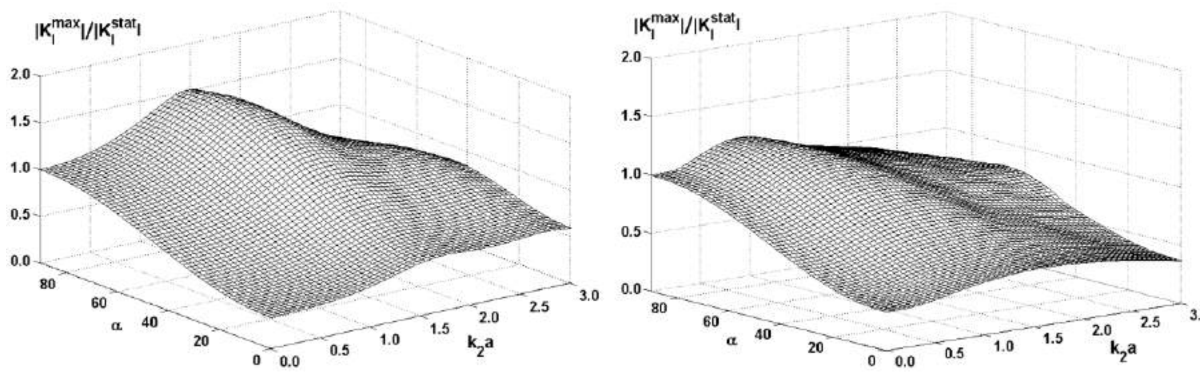

Figure 1: Stress intensity factors obtained neglecting the crack closure (left) and taking it into account (right).

The present study is devoted to application of boundary integral equations to 2D and 3D problems for interface cracks under impact and time-harmonic loading. The system of integral equations for displacements and tractions is derived from the dynamic Somigliana identity and the problem is solved numerically by the boundary elements method [1–3]. In order to take the crack closure into account the Signorini constraints and the Coulomb friction law are imposed. The considered non-linear contact problem requires an iterative solution procedure, which mutually solves the Neumann and Dirichlet linear problems in the cracked solid in combination with projections onto subsets of admissible displacements and contact forces. During the iterative process, the solution changes until the distribution of physical values satisfying the contact constraints is found [2]. The distributions of the displacements and tractions at the bimaterial interface are obtained and analysed. The dynamic stress intensity factors are computed as functions of the parameters of the incident loading and properties of the material. The results are compared with those obtained neglecting the crack closure.

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EXPERIMENTAL STUDY OF ROTOR VIBRATIONS

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ABSTRACT

We introduce a two-degrees-of-freedom, non-smooth model of a Jeffcott rotor with a bearing clearance. During operation, the rotor makes intermittent contact with a snubber ring with viscoelastic support. This configuration gives rise to a rich and complex dynamical behaviour, which is explored both theoretically and experimentally. This work extends the previous investigations undertaken by the CADR within University of Aberdeen to gain a deeper insight into the system response [1-4]. To this end, careful numerical simulations are carried out and then compared with experimental data obtained from a rig (Fig. 1(a)) previously constructed and designed at the CADR.

(a) (b) (c)

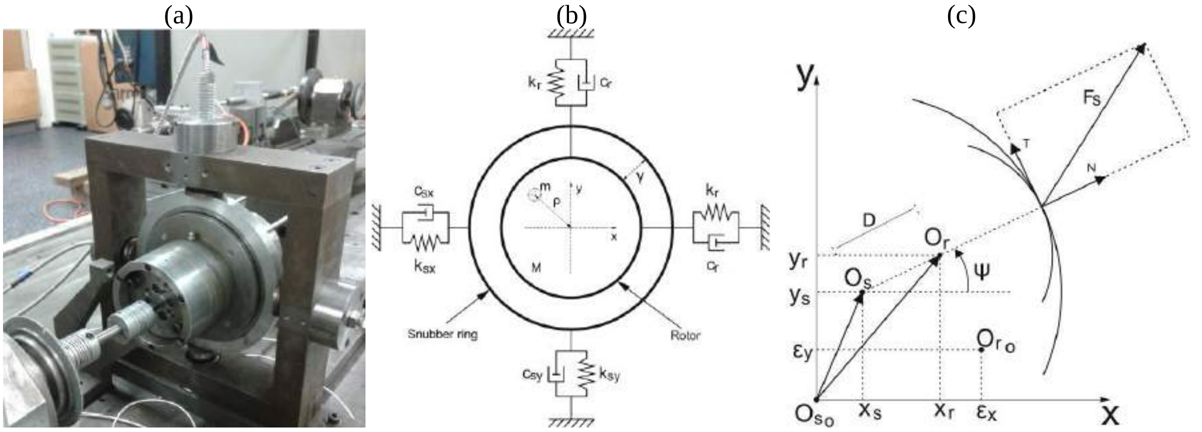

Figure 1: (a) Photograph of the experimental setup (b) Physical model of a Jeffcott rotor with a snubber ring. (c) Geometrical representation of the force produced by the snubber ring when an impact occurs.

The mathematical description of the system [5] is made under the following assumptions: the snubber ring is massless and with viscoelastic support; there is no dry friction between the rotor and the snubber ring; there are no gyroscopic forces acting on the system; and the gravity force is neglected. To derive the equations of motion based on the proposed model shown in Fig. 1(b) , we choose the coordinate system shown in Fig. 1(c), centered at the static position of the snubber ring O_{s0}.We denote by $(\varepsilon_x, \varepsilon_y)$ the eccentricity between the centre of the equilibrium position of the rotor O_{r0} andO_{s0}. Due to the physical configuration of the system, we must have D $\leq \gamma$ where γ denotes the radial clearance between the rotor and the snubber ring. Therefore to derive the equation of motions, we consider two regimes: No contact between rotor and snubber ring (D $< \gamma$) and contact between them (D$= \gamma$).

After a careful identification of the physical parameter values of the rig, we carry out a number of experiments which are then compared with the response of the mathematical model. In this way, a good agreement between the experimental and numerical results is achieved.

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APPLICATION OF THE BIFURCATION THEORY OF NONLINEAR DYNAMICAL SYSTEMS AND NEW RARE ATTRACTORS

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ABSTRACT

New applications of the so-called *bifurcation theory* [1-2] for forced nonlinear dynamical periodic systems (NDS) with several degrees of freedom are presented. Among them are a nonlinear mechanical system with double- and several-well potential, pendulum-like systems, and systems with impact interaction [3]. The bifurcation theory is established for direct global complete bifurcation analysis for essentially nonlinear dynamical periodic systems, described by models of ODE equations or by map-based models with discrete-time equations. Our approach is based on ideas of Poincaré, Andronov and many other modern scientists, working in global nonlinear dynamics, structural stability and bifurcations, topological and chaotic responses.

The main idea of the new bifurcation theory (BT) goes from a fact that the NDS in a given parameters and state spaces has finite number (usually not so many) of independent bifurcation groups S(p) with their own complex topology and bifurcations, chaotic behavior, and as a rule, with rare regular and chaotic rare attractors (RA). For each point of parameter space it is possible to find all stable and unstable fixed points of the periodic orbits. This periodic skeleton, with stable and unstable orbits, allows to mark out the main and bifurcation groups and to start global analysis in parameter and state spaces using found orbit for continuation in parameter space. The main concepts of the BT are: complete bifurcation group (BG), unstable periodic infinitiums (UPI-subgroups), responsible for chaos in the system; complex protuberances, and periodic skeletons for a system with parameter p or for a some restricted parameter space. For realization of the bifurcation theory in applications we use our software Spring [2].

For illustration of the advantages of the new bifurcation theory we use in this paper several typical nonlinear models, mentioned above, with one, two and several DOF. Special attention is paid for building complete bifurcation groups of the dynamical systems with multiplicity regions, chaotic attractors and rare subharmonic attractors, asymmetry and complex topology of their basins of attractors. Some open problems concerning using of the method of complete bifurcation groups and the bifurcation theory for new applications are also planned to discuss.

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NONLINEAR DYNAMICS, CHAOS AND CONTROL OF SMART MATERIAL SYSTEMS

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ABSTRACT

Smart material systems and structures have remarkable properties responsible for their applications in different fields of human knowledge. Shape memory alloys, piezoelectric ceramics, magnetorheological fluids, and magnetostritive materials constitute the most important materials that belong to the smart materials category.

Shape memory alloys (SMAs) are metallic alloys usually employed when large forces and displacements are required. Applications in aerospace structures, rotordynamics and several bioengineering devices are common nowadays. In terms of applied dynamics, SMAs are being used in order to explore adaptive dissipation associated with hysteresis loop and the mechanical property changes due to phase transformation.

This paper presents a general overview of nonlinear dynamics, chaos and control of smart material systems built with SMAs. Vibration absorbers, impact systems and structural systems are of concern [1-4]. The idea is to investigate the nonlinear dynamical behavior of systems described ordinary differential equations. Figure 1 presents some systems discussed in this work.

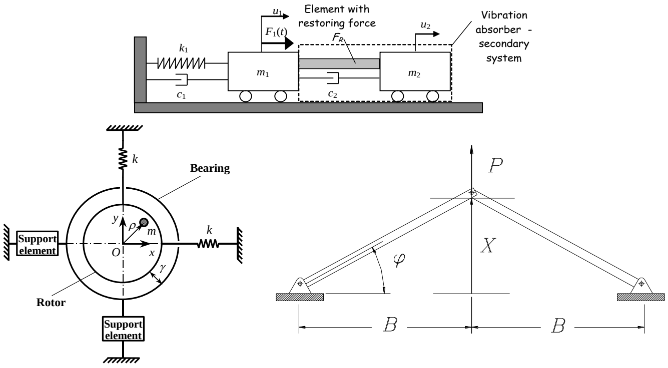

Figure 1: Schematics of SMA dynamical systems.

Chaos may be exploited in order to design dynamical systems that may quickly react to some new situation, changing conditions and their response. In this regard, the idea that chaotic behavior may be controlled by small perturbations allows this kind of behavior to be desirable in different applications. The application of chaos control in structural systems can be employed either to stabilize unstable periodic orbits embedded in chaotic attractor or to perform bifurcation control [5-6]. Hence, the combination of chaos control and the adaptability of smart material systems are essential points related to the conception of new bio-inspired systems. Here chaos control is applied to SMA two-bar truss in order to illustrate some application related to this central idea.

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REDUCED ORDER MODELS FROM HIGH DIMENSIONAL FRICTIONAL HYSTERESIS

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ABSTRACT

Hysteresis is observed in many systems, including materials with internal dissipation [1] and magnets [2]. The physics of hysteresis is complicated and highly nonlinear, with sharp corners at points of load reversal. Low dimensional evolution equations that model hysteresis are empirical, such as the famous BoucWen model (see [3], and references therein).

In this work, we develop a family of reduced order models for hysteresis, beginning with a highdimensional frictional system of the form

$$\mu \operatorname{sgn}(\dot{x}) + Kx = b\,f(t)\,, \tag{1}$$

where x is a high dimensional state (say, 500 dimensional); K is a symmetric positive definite matrix; b is a vector; μ is a diagonal matrix; $f(t)$ is a scalar external forcing; and the signum function "sgn" is understood to be plus 1 for positive values of the argument, minus 1 for negative values of the argument, and multivalued (within $[-1,1]$) when the argument is zero. Note that Eq. (1) is not a system of differential equations in the usual sense, and solution requires knowledge of the rate of change of $f(t)$. A well known method of solving such systems uses the Linear Complementarity Problem [4], which we have used and will discuss briefly for completeness. Our work begins with solutions of the above system for random matrices μ, K, and b, and oscillatory $f(t)$ with both large and small reversals in direction (i.e., not pure sinusoids).

Our numerical results show hysteresis. In particular, both major and minor hysteresis loops are found, similar to those observed experimentally in both plasticity as well as magnetism. Finally, the singular value decomposition (SVD, also known as the proper orthogonal decomposition or POD [5]) suggests that a *two* dimensional description of the data would capture most of the interesting aspects of the solution. However, it is not clear that a usefully accurate two-state reduced model can be rationally obtained from the original governing equation so as to predict these hysteresis results; or even how *any* reduced order model should be obtained in the first place, given the high dimensionality and the strong nonlinearity of the system.

The rest of the paper develops a systematic approach for developing reduced order models from such a high dimensional system. Several topics must be addressed before a reasonably accurate reduced order model is obtained. We consider choices of basis vectors to project on (those obtained from the SVD are not enough, and we also include basis vectors corresponding to directions where slip occurs more easily). We present a serendipitous analytical approximation for the term involving the signum nonlinearity, without which progress would be next to impossible. To obtain the slip direction in the space of generalized coordinates, we address a variational problem invoking maximal dissipation that leads to nonstandard and nonlinear eigenvalue problems (which, again, we are fortunately able to solve completely). A final evolution equation is obtained for a few states (say, 4 to 7) that gives quite acceptable numerical hysteresis results.

Our results are of academic interest because of the novel approach and application. The actual reduced order model obtained is presently more complicated than the highly simple BoucWen model, suggesting that empirical models of hysteresis may remain useful. Yet, our work both provides fresh insights as well as access to internal ("unmeasured") states that are neither incorporated in typical empirical models nor available in experiments. The present approach also captures minor loops in the hysteresis, which the BoucWen model does not. Finally, it is possible that future work may simplify and improve our approach, making it more competitive for routine computational applications. In this way, our work opens up interesting lines of new research into the mathematics and physics of hysteresis.

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ROTATING A PENDULUM WITH A NONLINEAR EXCITATION

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ABSTRACT

Attractors for a rotating pendulum subject to parametric excitation had been identified analytically and numerically, for example by Lenci *et al* [1] as well as Clifford and Bishop [2]. Although periodic rotating orbits have been predicted in parameter space, such basins of attraction were smaller, or even difficult to find, in experiment, as was observed by Yokoi and Hikihara [3], or in the work of Lenci and Rega [4]. A practical application to harness energy from sea waves requires a basin of sufficient size. Since a corresponding experimental basin is expected to be smaller, a practical and reliable means of initiating and maintaining stable rotations is needed. The authors would like to report that stable rotations, tumbling chaos, period-1 and period-2 oscillations have been created by constructing an experimental rig (see Fig. 1) consisting of pendulum suspended on a linear elastic spring. A ferrous conductor is mechanically connected to a point of suspension of pendulum, and travels vertically in linear guides within a solenoid. A series RLC circuit powers this solenoid from an alternating current supply, and is switched on and off periodically by means of solid state relay and function generator.

(a) (b)

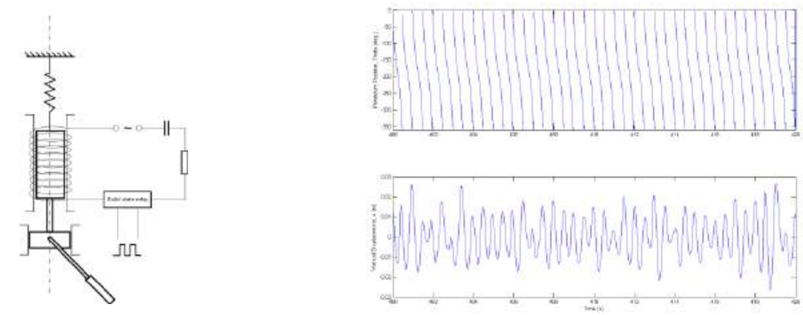

Figure 1: (a) Schematic of experimental rig to suspend pendulum with linear elastic spring to force with nonlinear electromagnetic attraction. (b) Experimental time histories or period-1 rotations effected by a nonlinear actuation.

A linear reluctance motor had been reported previously by Mendrela and Pudlowski [5], and is, in this abstract, a nonlinear displacement-dependent forcing parametrically exciting a pendulum. In relation to the work on electromagnetic interactions between pendulum and modal exciter [6], the current experimental rig's vertical motion is different because it is not prescribed to move per some intended waveform. Moreover, the pendulum experiences an electromagnetic force depending on (a) its position within solenoid and (b) instant in time when the circuit is switched on. The authors envisage this to be a useful method of control if frequency of solid state relay were equal to or twice the frequency of heaving waves. If pendulum were floating on waves, solenoid could be switched on and off selectively to complement wave displacement so as to steer towards a rotational attractor. A phase of forcing in relation to wave motion could also be adjusted to ensure an optimal transfer of energy [7].

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EVALUATION OF NONLINEAR SEISMIC RESPONSES OF FULL-SCALE INSTRUMENTED BUILDINGS

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ABSTRACT

In-situ measured responses of full-scale structural systems, such as bridges, buildings, dams, and off-shore installations, are influenced by all physical properties of the structure, foundations, soil and surrounding environment. They therefore represent ground truth about their performance and condition, and can help the researcher to free themselves from simplifying assumptions inherent to numerical models and laboratory scale experimentation. However, such investigations are still relatively rare in the existing literature. This study focuses on the analysis of nonlinear seismic responses of two buildings equipped with long term monitoring systems.

The first building is a three storey reinforced concrete (RC) structure located in Lower Hurt, New Zealand monitored for a period of more than two years (Fig. 1). System identification was used to extract dynamic modal properties under a selection of 50 earthquakes varying in magnitude from small to moderate. The extracted modal characteristics were examined statistically to develop relationships between response amplitudes (peak roof acceleration - PRA) and modal frequencies and damping ratios (Fig. 2). Nonlinearities in the building behaviour were clearly visible as decreasing trends of modal frequencies vs. PRA. A parametric finite element (FE) model of the building was formulated including also non-structural elements (cladding and glazing) and soil-structure interaction, which proved to significantly influence the dynamics. The parameters of the FE model were then tuned (updated) such that the model matched with high accuracy the experimentally observed frequency-PRA trends (Fig. 3). The updated FE model was used to assess the performance of the building at the serviceability limit state via time history simulations using a selection of scaled ground motions recorded at the building site. It was observed that the inter-storey drifts reached in several cases the recommended threshold values for the updated building model only, showing that without updating the performance assessment would have been unconservative and emphasizing the value of using the calibrated FE model.

The second analysed building is an eight-story RC building located in Christchurch, New Zealand. Acceleration data was available for several small magnitude seismic events in 2007, the devastating M_W 7.0 Darfield (04/09/2010) and M_W 6.3 Christchurch (22/02/2011) earthquakes and their aftershocks. It was found that strong nonlinearities in the structural response occurred and manifested themselves in all identified modal frequencies of the building that decreased by up to 39% when the building was shaken by the two strong earthquakes. Using the aftershocks, it was observed that a partial stiffness loss in the building appeared permanent as the natural frequencies remained by up to about 20% less than for the 2007 events. The investigation of peaks in the cross-spectra between the upper floor and foundation responses found the lowest natural frequencies to be appreciably influenced by nonlinear soil-structure interaction.

Fig. 1. 3-storey building.

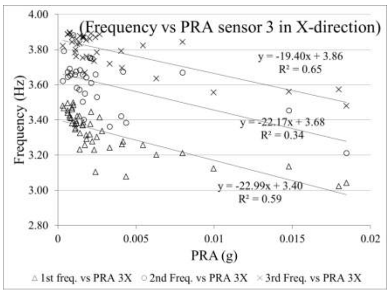

Fig. 2. Frequency-PRA trends.

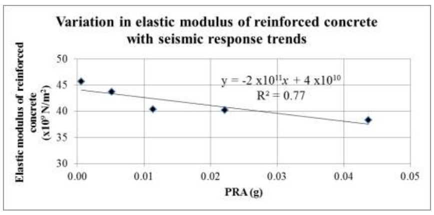

Fig. 3. Updated stiffness-PRA trends.

FREQUENCY-DOMAIN METHODS FOR VIBRATION-FATIGUE-LIFE ESTIMATION

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ABSTRACT

Vibrationally excited mechanical structures are exposed to vibration fatigue. Relating structural dynamics with fatigue opens new possibilities for the frequency-domain methods in the vibration fatigue. Frequency domain methods have been researched in the last decades and several numerical studies researched the performance of those methods. However, the older and the most recent methods are missing detailed experimental comparison side-by-side. The focus of the research is on the fatigue-life accuracy of the frequency-domain methods versus the time-domain approach. Experimentally 27 different Power-Spectral-Density profiles typical in structural dynamics and in automotive industry are researched. The methods, considered for comparison, are: empirical $\alpha_{0.75}$, Wirsching-Light, Gao-Moan, Tovo-Benasciutti (2 versions), Zhao-Baker (2 versions) and Petrucci-Zuccarello.

Real signal was acquired on an electro-dynamic shaker. The excitation was controlled according to different vibration spectra (*e.g.* one on Fig. 1). Fatigue-life estimates were calculated with presented frequency-domain methods and with the time-domain approach, which was considered accurate. The calculations were made for different materials, namely steel, aluminum and spring steel. Fig. 2 shows comparison of results for best performing methods on the automotive industry accelerated-vibration-test spectra.

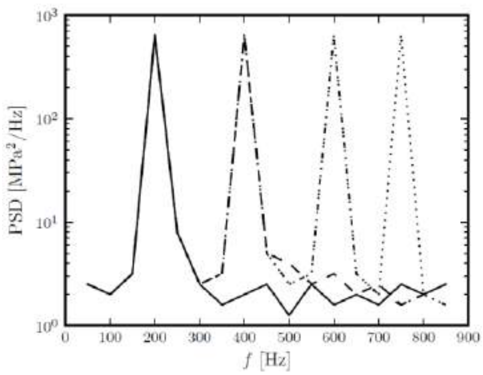

Figure 1: Typical multi-mode vibration power spectrum.

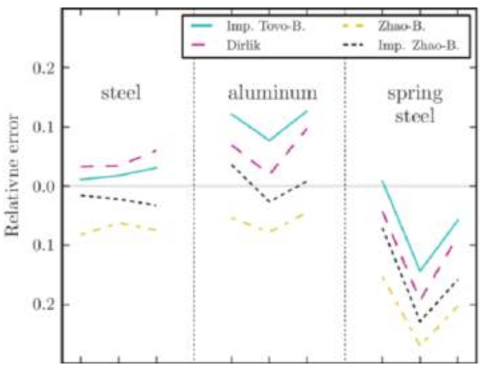

Figure 2: Comparison of results for different frequency-domain methods.

It was found that the Dirlik, Improved Zhao-Baker and Improved Tovo-Benasciutti methods gave most accurate results: fatigue life estimate was obtained consistently across different vibration profiles for materials with low values in the S-N slope.

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NONLINEAR MECHANICS AND DYNAMICS CHALLENGES FOR SUBSEA PIPELINES AND RISERS

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ABSTRACT

The author has more than twenty five years experience developing and applying numerical solutions to analyse the structural response of pipelines and risers in the offshore subsea environment. This paper will illustrate the current state-of-the-art of analysis techniques and design methodologies as they pertain to a range of key problems in the riser and pipeline industry. The paper is expected to touch on such areas as nonlinear riser dynamics [1,2,3], flexible pipe cross-section analysis [4], lateral and upheaval buckling of subsea pipelines [5], riser and pipeline vortex induced vibration response [6], and various scenarios of nonlinear contact modeling and pipeline installation analysis modeling. The paper will look at the use of both nonlinear finite element analysis and computational fluid dynamics methodologies.

To provide some further insight to the above, the author will present in outline some specific work on nonlinear beam mechanics for beams undergoing nonlinear large displacements under hydrodynamic loads and applied end motions – this work being very relevant to the analysis of risers and pipelines in the subsea environment. The theoretical work to be presented illustrates the mechanics of a beam element undergoing moderate deformation relative to a co-rotating set of axes. The moderate beam deformation equations are revealing in terms of how they illustrate the coupling between axial and lateral deformations and between bending and torque terms.

In the context of the background theory and range of problems mentioned above, the author will highlight the current challenges facing the industry. Given this and combined with the fact that this paper will have a strong focus on applications and design methodology, the author will provide conference delegates with an extensive basis for evaluating the potential application of their own analysis methodologies to the offshore subsea environment. The paper will provide an up-to-date insight on the analytical challenges facing the designers of offshore risers and pipelines.

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GLOBAL WORKOVER RISER ANALYSIS

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ABSTRACT

When performing operations, such as drilling and intervention etc., to a subsea well, riser system is required to connect between the subsea wellhead and the operating rig built on some form of vessel. The configuration of a typical top tensioned workover riser is shown in Fig. 1. At top of the wellhead, subsea tree packages are installed for the purpose of performing various operations. The bottom end of the riser is mounted to the top of subsea equipment, and the top of the riser is tied back to the vessel. Top tension is applied to the tension ring of the riser through tensioning system of the rig.

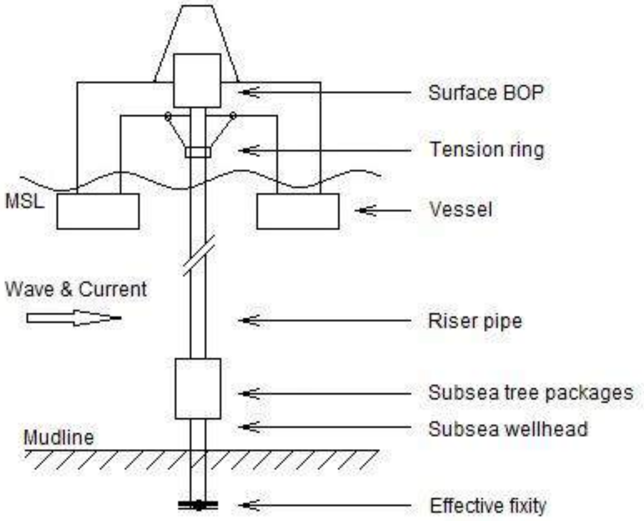

Figure 1: Schematic of a top tensioned workover riser system.

Flexcom, a nonlinear three-dimensional finite element package, is used to perform the modeling of the riser system. The riser system with complex geometries is simplified as a string of elements with descriptions of diameters, stiffness and inertia. The hundreds-meter conductor below mudline is modeled only by a section of conductor ended at an effective fixity, considering the limited effect to the riser and the complexity of modeling. Motions and states of the riser system in response to currents and waves are investigated based on the results from *Flexcom*. Interaction between the vessel and the riser string is neglected, since the huge conflict of weights between the riser and the vessel introduces negligible effect.

The analysis follows a number of procedures with guidance of regulations [1]–[3]. First of all, the optimized top tension is determined to 1) guarantee the riser purely in tension, and 2) obtain best performance. Strength of each element of the riser system is checked for various operating conditions in order to define an operating envelope. Assessment of vortex induced vibration (VIV) is performed, and the critical current range where VIV may occur is therefore located. Fatigue analysis is conducted as well to find out the service life of the riser system.

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QUASI-ROTATION METHOD FOR BEAMS UNDERGOING LARGE DEFLECTION WITH COUPLED TORSION, BENDING AND AXIAL DEFORMATION

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ABSTRACT

Nonlinear analysis of beams undergoing finite deflections and rotations has been the subject of extensive research with much attention given to the formulation of finite elements for moderate deformations and coupled torsion, bending and axial response [1-3]. However analysis of subsea beam structures demands that continued improvement is required regarding convergence rates for beam finite element analysis. This has lead the authors to develop a finite element beam definition which is based on the following:

1. the convected coordinates method;
2. a moderate beam deformation definition relative to the convected axis with coupling of axial, bending and torque response;
3. a description of the beam moderate rotations in terms of a set of varying twist-bend angles along the beam element;
4. the use of rate of rotation and quasi-rotation definitions to track the path dependent nature of rotations in three dimensions
5. a Newton-Raphson solution scheme

The quasi-rotation vector quantity and its time derivative rate of rotation in space $\dot{\boldsymbol{\omega}}$, is shown by the authors to be related to a rotational deformation quantity for a beam where rotational deformation along the beam is defined by a set of varying twist-bend angles with respect to moving axis system denoted a convected coordinates axis.

The relationship described above allows for the definition of generalized beam strain measures, with full term coupling, which leads to a finite element solution with a much reduced set of solution terms, by comparison to other solution methods. The rate of rotation measure which forms the basis for strain terms is given as follows:

$$\dot{\boldsymbol{\omega}} = \left(1 - \frac{\beta^2}{2}\right)\frac{d\theta_t}{dx}\boldsymbol{e}_c + \frac{1}{2}\left((\boldsymbol{e}_c \times \boldsymbol{t}) \cdot \frac{d\boldsymbol{t}}{dx}\right)\boldsymbol{e}_c + \beta\frac{d\theta_t}{dx}(\boldsymbol{n} \times \boldsymbol{e}_c) + \frac{d}{dx}(\boldsymbol{e}_c \times \boldsymbol{t})$$

where

$\boldsymbol{e}_c$	unit vector parallel to the element convected coordinate axis
$\boldsymbol{n}$	unit vector normal to the bend plane of the element
$\boldsymbol{t}$	tangent to deformed beam centreline
x	beam convected axis joining beam end nodes
β	bend angle between $\boldsymbol{e}_c$ and $\boldsymbol{t}$
θ_t	twist angle about either $\boldsymbol{t}$ and $\boldsymbol{e}_c$

The paper demonstrates the development of the strain terms and the Newton Raphson finite element formulation and illustrates, through independent validation, how the finite element solution when applied to highly nonlinear subsea riser problems, shown high accuracy and improved solution speeds compared to existing analysis methods.

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Day 2: 22 August 2013

Brazilian Session (8:00 – 10:20)

“*Nonlinear dynamics of atomic force microscopy”,* G Rega, U Andreaus, L Placidi, V Settimi

“Using feedback control for discovery”, J Sieber, D A W Barton

“The influence of imperfections and uncertainties in nonlinear structural dynamics”, P B Gonçalves

“Phase synchronization of coupled small-world neuronal networks with short-term synaptic plasticity”, F Han, B Zhang

“Nonlinear interactions between unstable vibartion modes of a fluid-conveying pipe”, K Yamashita, H Kosaki, H Yabuno

“Nonlinear dynamics of thermo-visco-elastic cantilever sensor arrays”, O Gottlieb, F Torres, T Mintz, G Vidal, E Hollander, N Barniol

Indian Session (10:40 – 12:40)

“Dynamic analysis of tank container handling with arbitrary filling level”, A Hansen, E Kreuzer, M A Pick, C Radisch

“Stress-free layers in photoinduced deformations of beams”, Gy Károlyi, Z Ábrahám

“Nonlinear dynamics of a mechanical oscillator coupled to an electro-magnetic circuit”, I T Georgiou, F Romeo

“Numerical optimization of Duffing oscillator”, P Brzeski, P Perlikowski, T Kapitaniak

“Rotation disturbed by second harmonic excitation in parametrically excited pendulum”, Y Yokoi, T Higuchi

“A new normal form method for analysing systems of coupled nonlinear oscillators”, T Hill, A Cammarano, S A Neild, D J Wagg

NONLINEAR DYNAMICS OF ATOMIC FORCE MICROSCOPY

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Atomic Force Microscopes (AFMs) are powerful devices used for surface analysis in nano-electronics, mechanics of materials and biotechnology, as they permit to topologically characterize surfaces up to micro and nano resolution levels. In a typical AFM, the topography is imaged by scanning a sharp tip, fixed to the free end of a microcantilever vertically bending over the sample surface, and by measuring the tip deflection through a laser technology. The tip-sample interaction modifies the beam dynamics and allows not only to image surfaces, but also to measure some physical properties of the sample. The most common operation modes of AFMs are the noncontact mode, in which there is absence of contact between the tip and the sample, and their interaction is governed by a solely attractive potential, and the tapping mode, in which the tip operates in both attractive and repulsive force regions and touches the surface only for short time intervals. In this lecture, both operation modes are addressed via a continuous beam model, with the interaction force being accounted for as a localized nonlinear field force. However, different continuous and Galerkin-reduced models are considered, and various aspects of nonlinear dynamics, typical of the two different AFM systems, are investigated and discussed in a somehow complementary way.

In the noncontact AFM, whose cantilever tip has to maintain a design gap from the sample to ensure that the beam elastic restoring force is stronger than the atomic attraction, interest is mostly towards investigating conditions for possible occurrence of the unwanted phenomenon of jump to contact (escape), in phenomenological (dynamical) terms. To this aim, attention is systematically devoted to the strongly nonlinear dynamics of the system under beam vertical or scan horizontal excitation, which realize conditions of external or parametric resonance, respectively, by considering the relevant primary (fundamental) and subharmonic (principal) resonances of the dominant beam mode. Given the width of the bifurcation analyses, a minimal-order (single-mode) model of beam is considered, yet with the underlying continuous model consistently incorporating geometric nonlinearities, nonlinear atomic interaction, and generalized forces describing motion control of the microcantilever. In this way, a general platform to possibly conduct successive, more refined, multimodal investigations also accounting for the nonlinear coupling effects is realized.

The nonlinear dynamic behavior of the single-mode model is analyzed in terms of attractors robustness and basins integrity. Local bifurcation analyses are carried out to identify the overall stability boundary in the excitation parameter space as the envelope of system local escapes. The dynamic integrity of periodic bounded solutions is studied, and basin erosion is evaluated by means of different integrity measures. The ensuing erosion profiles allow us to dwell on the possible lack of homogeneous safety of the stability boundary in terms of attractors robustness, and to identify practical escape thresholds ensuring an a priori design safety target.

In contrast, in the tapping mode AFM, the interest is devoted to highlighting the effect of also higher order eigenmodes, with the relevant damping ratios, on the overall system dynamics, which is indeed addressed via a multimode approximation allowing to consider external excitation at primary or secondary resonance of different modes. In this case, a simple linear beam model is considered, with the dynamic response being investigated via numerical simulations of up to a three-mode reduced model, which appears indeed sufficient to catch the main nonlinear dynamic phenomena. Different bifurcation parameters are considered, namely the excitation frequency and the approach/retract separation between cantilever and sample. Typical features of tapping mode AFM response as nonlinear hysteresis, bistability, higher harmonics contribution, impact velocity and contact force are addressed. The analysis is conducted by evaluating damping of higher modes according to the Rayleigh criterion, which basically accounts for structural damping representative of the behavior of AFMs in air. However, nominal damping situations more typical of AFMs in liquids are also investigated, by considering sets of modal Q-factors with different patterns and ranges of values, and comparing the relevant responses. Variable attractive-repulsive effects are highlighted, along with the possible presence of a coexisting multi-periodic orbit when the system is excited at second resonance. The importance of considering excitation of also the second mode to the aim of evaluating possibly harmful tapping effects on the sample is discussed.

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USING FEEDBACK CONROL FOR DISCOVERY

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ABSTRACT

Most of the time feedback control is used to manipulate a dynamical system in order to force it into showing a desired behaviour. For example, one feeds back a signal $u(t)$ depending on the difference between some output $y(t)$ and some demand (or reference) signal $y_{\mathrm{ref}}(t)$ with the goal that $y(t)$ matches the a-priori given $y_{\mathrm{ref}}(t)$ as closely as possible (in a tracking task). However, one can also use feedback control to discover natural trajectories of the uncontrolled system by gradually adjusting y_{ref} to make the difference $y(t)$-$y_{\mathrm{ref}}(t)$ vanish. This idea was introduced by physicists under the term *non-invasive control*. Commonly schemes are was-out filters and time-delayed feedback (setting $y_{\mathrm{ref}}(t)=y(t-T)$) [1]. However, both of these schemes have difficulties in the most common scenario for instability. They can't find and stabilise equilibria or periodic orbits next to a fold (saddle-node bifurcation). Several demonstrations have shown that, in principle, the appropriate procedure for adjusting y_{ref} is a Newton iteration [2]. However, if one wants to find a bifurcation diagram with respect to a parameter p and the feedback control is introduced through modification of p. Say, we want to explore a family of equilibria near a fold in the parameter p, and introduce feedback via

$$u(t)=p_0+g\,(y(t)-y_0),$$

where g is the control gain. As Figure 1(a) shows, regardless of our choice of p_0 and y_0, the feedback controlled system will asymptote to a natural equilibrium of the uncontrolled system, namely, at the parameter value $p_1=\lim_{t\to\infty} u(t)$, with the output $y_1=\lim_{t\to\infty} y(t)$.

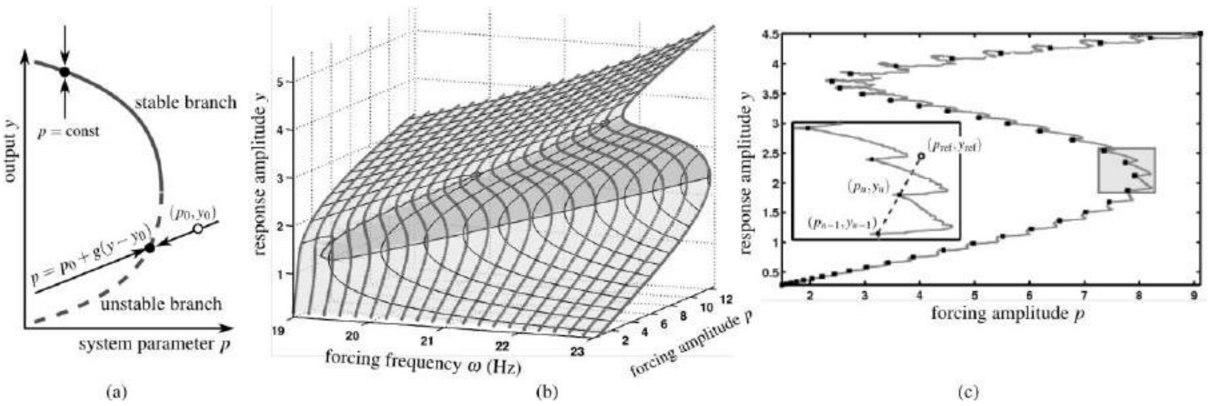

Figure 1: (a) Schematic, illustrating the idea for finding natural unstable equilibria. (b) Experimental bifurcation diagram of electro-magnetic energy harvester. (c) Illustration, how the controlled system moves through the diagram.

Figure 1(b) shows a complete experimental bifurcation diagram of periodic orbits in the frequency-amplitude plane for an electromagnetic energy harvester [3]. In this case, output $y(t)$ and reference $y_{\mathrm{ref}}(t)$ were periodic and the control was a (filtered) PD control. Fig 1(c) shows how the system "moves through the bifurcation diagram" in time. Rapid adjustments in the *x*-direction (forcing amplitude) due to a new setting of the parameters are followed by slower asymptotes where the control stabilises the system toward its steady state.

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THE INFLUENCE OF IMPERFECTIONS AND UNCERTAINTIES IN NONLINEAR STRUCTURAL DYNAMICS

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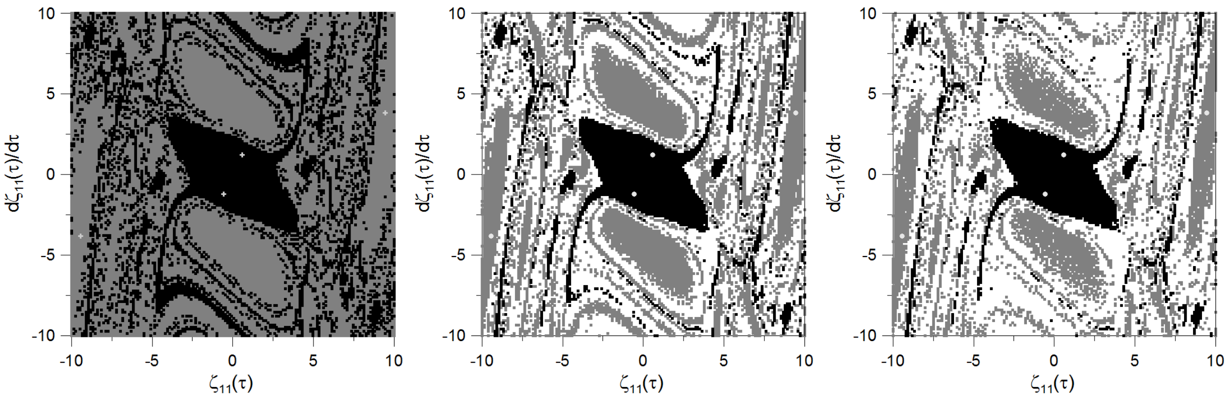
ABSTRACT

Slender structural systems, particularly those liable to unstable buckling, usually become unstable at load levels lower than the linear buckling load of the perfect structure. In some cases as, for example, cylindrical shells under axial compression, experimental buckling loads can be just a small fraction of the theoretical critical load. This is mainly due to the imperfections present in real structures, being geometric imperfections the main culprit in slender structures. The imperfection sensitivity of structures under static loading is well studied in literature [1], but little is known on the sensitivity of these structures under dynamic loads. In a dynamic environment not only geometric imperfections but also initial conditions (disturbances), physical and geometrical system parameters uncertainties and excitation noise influence the bifurcation scenario [2, 3]. Finally, mathematical models for many engineering systems involve several symmetries and breaking some of the underlying symmetries may also lead to significant modifications of the nonlinear system response [4]. The aim of this work is to discuss the influence of inherent imperfections and uncertainties of real systems on the dynamic integrity and stability of their solutions in a dynamic environment. To illustrate the system sensitivity, results involving structural elements such as bars and shells and some simplified archetypal models of slender systems liable to buckling are used. Figure 1 exemplifies the increasing sensitivity of a cylindrical shell under axial harmonic load plus random noise ($\Gamma = \Gamma_0 + \Gamma_1 \cos(\omega t) + G(\Gamma_1, \omega, t)$). The black and grey regions correspond to pre-buckling, small amplitude, periodic attractors and post-buckling, large amplitude, periodic attractors, respectively, while the white region corresponds to sets of initial conditions sensitive to random noise (the final outcome is sensitive to random noise – frequency bandwidth and standard deviation parameter).



(a) $\omega_1 = 0.0$, $\delta = 0.00$ (b) $\omega_1 = 0.25$, $\delta = 0.05$ (b) $\omega_1 = 0.25$, $\delta = 0.10$

Figure 1. Cross sections of the basin of the attraction of the shell submitted to (a) a deterministic and (b, c) non-deterministic load. ($\Gamma_0 = 0.40$, $\Gamma_1 = 0.40$, $\Omega = 1.60$). $\omega_1 =$ frequency bandwidth of the excitation frequency, rad/s; $\delta =$ standard deviation parameter.

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PHASE SYNCHRONIZATION OF COUPLED SMALL-WORLD NEURONAL NETWORKS WITH SHORT-TERM SYNAPTIC PLASTICITY

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ABSTRACT

The connection architecture of human brain, which is composed of billions of neurons connected to each other via synapses, is very complicated. However, neuroanatomic studies reveal that neurons with similar connectional and functional features are grouped into clusters with 10^5 to 10^6 cells with spatial localization. Such clusters form structures called cortical areas or subcortical nuclei [1]. Although the real structures of these clusters are not clear yet, small-world connectivity has been found in some biological neuronal networks [2]. Thus it might be useful for us to investigate the collective dynamics of neurons on coupled small-world neuronal networks.

Chaotic bursting is a fundamental behavior of neurons. Synchronization is thought to play an important role in information processing of human brain. So we define phase synchronization (PS) of bursting neurons [3] and study PS for the neuronal networks in this paper. Here, the neuronal network consists of two subnetworks, each of which is a small-world network, and the two subnetworks are connected by random connections with an inter connection probability. As in biological neuronal networks, the weights of synapses keep changing in the growth of cells and in studying and memorizing processes of the brain, we also consider synaptic plasticity, which is described by a learning rule [4]. Then PS of the coupled small-world neuronal networks is studied carefully. First, we explore local PS for the two subnetworks and then investigate the critical coupling strengths for global PS of the entire network with different inter connection probabilities. Second, we study the effect of intra and inter connection probabilities on PS, respectively, and obtain the optimal value for the two parameters so as the network achieves global PS most easily. Third, we investigate the relationship between PS and the synaptic plasticity. Finally, we plot the spatial patterns to demonstrate our findings and find out more interesting phenomena.

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NONLINEAR INTERACTIONS BETWEEN UNSTABLE VIBARTION MODES OF A FLUID-CONVEYING PIPE

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ABSTRACT

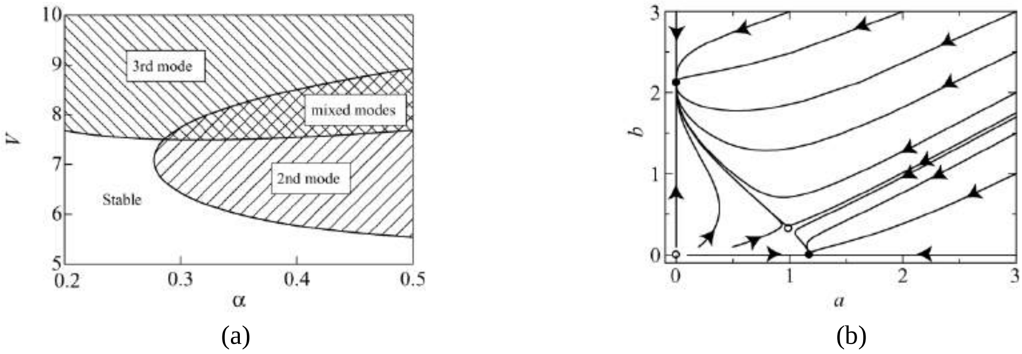
A certain mode of the lateral pipe vibration is self-excited when the flow velocity V exceeds its critical value V_{cr}. In the range of the flow velocity slightly above the critical value V_{cr}, an evolutional equation of the complex amplitude for an unstable mode was derived and the finite amplitude of the self-excited vibration was determined [1]. The self-excited pipe vibration becomes chaotic as the flow velocity V is further increased compared with the critical flow velocity V_{cr}. Copeland and Moon experimentally clarified that an attached end mass α to the cantilevered pipe system causes many complex dynamical motions between the occurrence of self-excited vibrations and the chaotic motions [2]. This study deals with the planar pipe vibrations in the case that two distinct eigen modes of the pipe are simultaneously self-excited.

Figure 1 (a) shows the linear stability boundaries of self-excited pipe vibrations in α-V plane. For large end mass, second and third modes could be simultaneously destabilized in a certain range of the flow velocity. Moreover, the lateral deflection v of the pipe is assumed as $v = A(t)\ \Phi_2 \exp(i\omega_{2r}t) + B(t)\ \Phi_3 \exp(i\omega_{3r}t)$ +c.c., where Φ_j and ω_{jr} (j=2,3) are the eigen function and the eigen frequency of the jth mode, respectively. By considering the orthogonal conditions between the eigen functions Φ_j and their adjoint functions, evolutional equations of complex amplitudes A and B are derived from the nonlinear nonself-adjoint differential governing equation and boundary condition.

$$\dot{A} = -\omega_{2i}A + \xi_{12}|A|^2A + \xi_{22}A|B|^2 \quad (1)$$

$$\dot{B} = -\omega_{3i}B + \xi_{13}|B|^2B + \xi_{23}B|A|^2 \quad (2)$$

where ω_{ji} (j=2,3) is the damping ratio of the jth mode.



(a) (b)

Figure 1: (a) Linear stabilities of self-excited pipe vibrations (/, unstable in second mode; \, unstable in third mode; ×, unstable in second and third modes. (b) Flow in the vicinity of the steady state solutions (V = 8.0, α = 0.36)

After letting A=$a$exp(iϕ)/2 and B=$b$exp(iψ)/2, the nonlinear interactions between the second and third modes are clarified by numerically solving the amplitude equations. Figure 1 (b) shows flow in the vicinity of the steady state solutions (V = 8.0, α = 0.36), where ∘ and · indicate the stable and unstable steady state solutions, respectively. As shown in Fig. 1 (b), mixed modal self-excited vibration ($a_s \neq 0$ and $b_s \neq 0$) is unstable. The self-excited pipe vibration may be produced either in second mode or third mode, depending on the initial conditions.

The experiments were conducted to verify the theoretical results. As predicted with the theoretical analysis, it was confirmed that the self-excited vibration in second or third mode can coexist within a certain range of the flow velocity, depending on the initial conditions.

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NONLINEAR DYNAMICS OF THERMO-VISCO-ELASTIC CANTILEVER SENSOR ARRAYS

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ABSTRACT

Arrays of nano- and micro electromechanical cantilevers have been proposed over a decade ago for fast imaging in atomic force microscopy [1] and more recently for multiple detection of mass sensing [2]. To allow for increased sensitivity and robustness of measurements it is crucial to understand the nonlinear dynamics of these arrays as their resonant dynamics is no longer small, and can exhibit a complex bifurcation structure that includes synchronous periodic or quasiperiodic response, and de-synchronized chaotic dynamics [3]. However, to-date, there is no quantitative comparison between theory and experiments of micro- and nano- cantilever sensor arrays. The lacking of consistent theoretical models that are experimentally validated in both linear and nonlinear regimes of operation, do not enable robust prediction of response. This is of particular importance for electro-mechanical sensor arrays in high vacuum where response is nonlinear and non-stationary due to strong thermo-visco-elastic coupling of the individual array elements. We thus investigate an electrodynamic array of coupled conductive cantilevers which can be excited externally or parametrically via alternative configurations of electrodes. Calibration of individual element material properties is done via a novel asymptotic model-based estimation procedure which enables identification of a hardening cubic stiffness nonlinearity and cubic thermo-visco-elastic damping coefficients [4]. The latter is obtained from a parametric excitation configuration where linear damping governs the frequency response bandwidth and the maximal response is governed by cubic damping (Figure 1a). A numerical investigation of a three element array yields periodic solutions with both in-phase and out-of-phase synchronization between the elements. A decrease in system damping reveals loss of orbital stability culminating with quasiperiodic dynamics (Figure 1b) due to a 3:1 internal resonance.

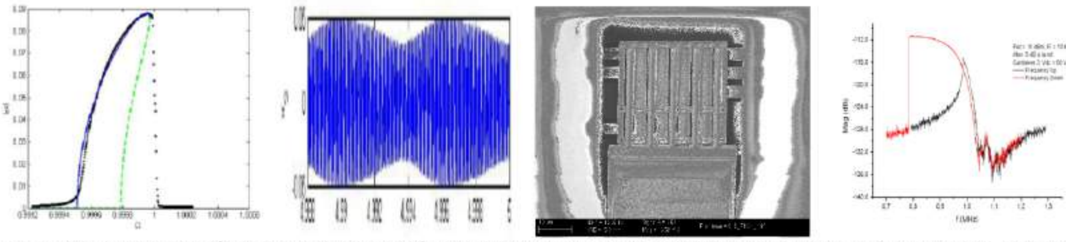

Figure 1: (a) Asymptotic frequency response of a parametrically excited individual element, (b) Numerical quasiperiodic dynamics in a three element array, (c) five element CMOS cantilever array, (d) Experimental frequency response.

In order to validate the onset of de-synchronization we make use of a conventional complementary meta-oxide semiconductor (CMOS) process [5] recently employed to manufacture individual cantilever sensors with attogram resolution to manufacture a five element array (Figure 1c) where coupling between elements is achieved via a common overhang at the base of the microcantilevers. External excitation of the array via a set of bottom electrodes reveals bi-stable behavior of the far right element (Figure 1d). The influence of individual element coupling (e.g. depth of overhang) and operation in vacuum (e.g. thermo-visco-elastic damping) on orbital stability, synchronization and array sensitivity will be discussed.

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DYNAMIC ANALYSIS OF TANK CONTAINER HANDLING WITH ARBITRARY FILLING LEVEL

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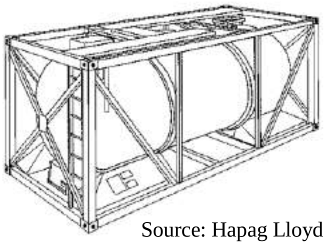
ABSTRACT

The majority of non-bulk cargo worldwide is transported by means of containers. Fast and safe container handling is, therefore, an important link in the supply chain. Research on the dynamics of container cranes is motivated by the vast amount of cargo loaded in harbors around the world and the reduction of costs related to that. In particular, tank containers are used to carry liquids like chemical substances or food products. These have usually dimensions of a standard 20' sea container and carry a cylindrical tank for payloads of up to 26 t, Figure 1. Due to the unavoidable occurrence of liquid motion inside the tank there is a restriction of the filling level to be over 80% or below 20% whenever the container is transported [1].

A goal of the following considerations is to investigate the influence of the liquid motion, also referred to as liquid sloshing, on the container dynamics. Once the effects can be described by appropriate models, control strategies can be developed that support the operator, provide additional safety and allow the loading of containers with arbitrary filling level.



Source: Hapag Lloyd AG, Hamburg, Germany

Figure 2: Tank container

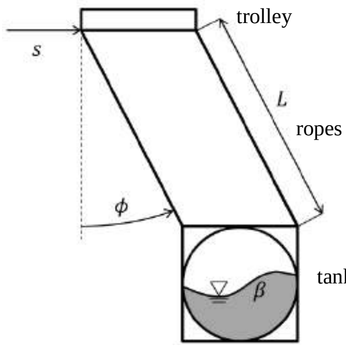


Figure 1: Load system model

First, liquid surface motion is modeled as a potential flow where the first resonant mode of the solution dominates the overall behavior when amplitudes are small [2, 3]. Given the acceleration of the container and the current state of the fluid motion, the sloshing forces that apply to the container can be computed. With this, the equations of motion for the load system with variable trolley position and rope length (see Figure 2) are developed in the second step. The resulting dynamic system is actuated by rope and trolley reference velocities that are to be established by internal servo controllers, which are modeled as first order systems. Measurements of current rope length and trolley position as well as the four individual rope forces are available. Finally, an optimal controller and observer design is applied to the linearized system and the behavior is tuned and evaluated by simulation of the controlled nonlinear system. The linear controller proves to be efficient in stabilizing the load at a given rope length and trolley position as well as in following reference trajectories while damping container oscillations and liquid sloshing. However, with a change in rope length the performance severely degrades due to the nonlinearity of the plant. To overcome this problem, a nonlinear controller is developed.

The *Institute of Mechanics and Ocean Engineering* runs a container crane test bench where controllers can be implemented and tested in scale 1:6 compared to real cranes. For this particular project the test bench is extended by a tank container filled with water to provide experimental results for validation of the presented model and control approaches.

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STRESS-FREE LAYERS IN PHOTOINDUCED DEFORMATIONS OF BEAMS

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ABSTRACT

Stiff, rodlike molecules constituting nematic liquid crystals align below the nematic-isotropic phase transition temperature, resulting in uniaxial orientational order. When rubber is formed from nematic liquid crystals by including a network of long molecular chains, the resulting *liquid crystal elastomer* becomes capable of large extension (up to 400 %) when taken through its nematic-isotropic phase transition temperature [1,2]. When the liquid crystal elastomer contains azo dyes (e.g. azobenzene) or other photoisomerizable molecular rods, under the effect of light, photon absorption implies a trans → cis transition and the shape of these rods become strongly kinked. This effect dilutes the nematic order and results in the contraction of the liquid crystal elastomer comparable in magnitude to that observed for thermal nematic-isotropic phase transition [3].
In the simplest approximation, according to Beer's law, as a result of photon absorption light intensity decays exponentially with the depth of the penetration [4]. Since light intensity governs the nematic order, the latter is also not uniform across the thickness of the elastomer film. As a consequence, a non-uniform photoinduced strain in *nematic photoelastomer* beams develops, decaying also exponentially with the depth. The induced stress will hence also be non-linear across the cross-section of the beam. It has been shown that there can be more than one stress-free layers within the cross-section of the beam [4,5].
In this talk, we offer a systematic dimensionless parametric study of nematic photoelastomer beams under the effects of incoming light and imposed mechanical loads (forces and torques). We show analytically how the number of stress-free layers depends on the parameters of the problem. The paths traced out by the system in the space of dimensionless parameters by varying the different main parameters is shown, which sheds light on how the number of stress-free layers changes when e.g. the thickness of the elastomer film is varied. We also investigate the case of optimal bending, that is, when largest curvature arises under light absorbtion. This is important for the applications, micropumps [6] or other devices to be applied in microfluidics [5,7], photomechanical actuators [5,8], manipulators of nanostructures [7] or artificial muscles [8].

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NONLINEAR DYNAMICS OF A MECHANICAL OSCILLATOR COUPLED TO AN ELECTRO-MAGNETIC CIRCUIT

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ABSTRACT

The dynamics of a nonlinear electro-magneto-mechanical coupled system is addressed. The nonlinear behavior arises from the coupling quadratic nonlinearities due to the dependence of the inductance on the displacement of the metallic oscillator mass. When the linear frequency of the circuit is larger than that of the mechanical oscillator, the dynamics exhibits slow and fast time scales. The dynamics of the system excited by a harmonic voltage is analyzed while the external forcing in the mechanical part is absent; when the forcing frequency is close to that of the mechanical oscillator, the long term damped dynamics evolves in a purely slow timescale with no interaction with the fast time scale. In a recent study [1], the existence of a slow invariant manifold [2] and its computation were shown for this system. Direct numerical simulations on both full- and reduced-order systems have highlighted several interesting phenomena: the nonlinear resonance related to the current quadratic nonlinearity which imposes a natural linear resonance at half the frequency of the linear oscillator; the pull-in phenomenon, denoted by a jump after which the mass of the linear mechanical oscillator is pulled at a large distance and it is forced to oscillate about it; the occurrence of irregular dynamics for high excitation amplitude, involving a number of bifurcations characterized by dramatic qualitative changes of both the mechanical and electrical responses.

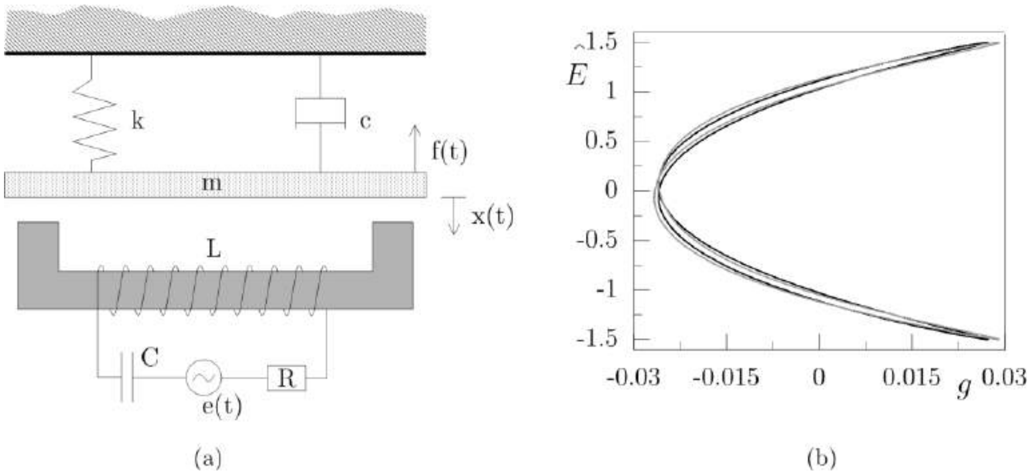

Figure 1: Schematics of the coupled system (a) and bifurcation diagram of the mechanical response (b).

In Figure 1a the electro-magneto-mechanical system, governed by equations (1), is schematically depicted.

$$\begin{aligned} &\ddot{x}+2\zeta_m\omega_m\dot{x}+\omega_m^2 x=\varepsilon\alpha\dot{q}^2+\hat{f} \\ &\left(1+\alpha x\right)\ddot{q}+\left(2\zeta_e w_e+\alpha\dot{x}\right)\dot{q}+\omega_e^2 q=\hat{e} \end{aligned} \qquad (1)$$

The reduced order systems, derived up to the 10^{th} order, enable to reproduce accurately the full system low frequency response, as shown by the comparison between the full-order (black) and the 10^{th} order reduced system (gray) mechanical restoring force. The developed numerical and analytical tools will be also used to give more insight into the rich bifurcation scenario.

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NUMERICAL OPTIMIZATION OF DUFFING OSCILLATOR

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ABSTRACT

In our previous paper we show dynamics of Duffing oscillator with suspended tuned mass absorber (TMA), we present complete bifurcation diagram in two parameters space (amplitude and frequency of excitation of Duffing system). In this paper we consider two different types of the TMA suspended on the forced Duffing oscillator (see Fig. 1) and we focus on the energy absorption properties of each attached system.

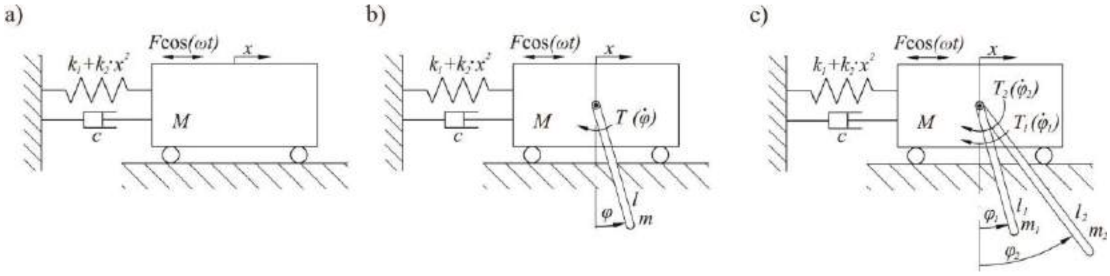

Figure 1: Model of system: Duffing oscillator (a), classical TMA with single pendulum (b) and TMA with dual-pendulum TMA (c).

A single pendulum is the classical TMA (Fig 1b) and dual-pendulum system is its modification (Fig 1c). In engineering one can find many successful applications of TMA both in mechanical engineering and in constructions (high buildings, long bridges etc.). Nevertheless, the optimization and appropriate selection of absorber parameters is a changeling task. As a

We present analysis of influence on Duffing systems amplitude of following parameters: damping in pivot of pendulums, masses of pendulums and lengths of pendulums. For those parameters we compute the frequency response curve for Duffing system and its L2-norm. The goal of our optimization is a minimization of L2-norm, so we want to achieve the lowest amplitude of Duffing oscillator in the considered range of systems excitation frequency (ω).

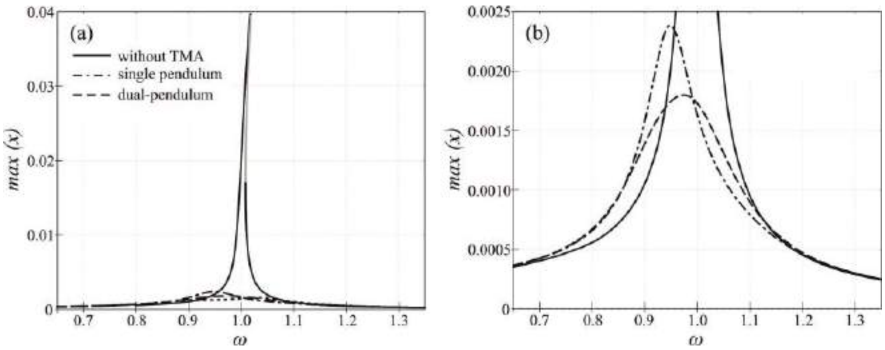

Figure 2: Frequency response curves for Duffing system without and with two types of TMA (a,b). Maximum amplitude of Duffing oscillator without TMA equals: 0.039, for simple one-pendulum TMA: 0.0024 and 0.0018 for dual-pendulum TMA. The black and gray lines correspond to stable and unstable periodic solutions. Changes of stability of periodic solutions occur thought the saddle-node bifurcations.

After optimization we observe a large decrease of Duffing system amplitude (see Fig. 2). The single pendulum TMA reduces it to 7% of maximum amplitude of Duffing without TMA and for dual pendulum system we observe the 6% reduction.

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ROTATION DISTURBED BY SECOND HARMONIC EXCITATION IN PARAMETRICALLY EXCITED PENDULUM

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ABSTRACT

Pendulum can convert vibration into rotation through its rotating motion under a vertical excitation. The converting motion has potential for energy extraction from nature. The conversion is based on the dynamics in the parametrically excited pendulum. In most studies for the dynamical system, a sinusoidal or single frequency wave is assumed as the parametric excitation. From the point of view of the application or a natural generalization, other waveforms are employed such as elliptical excitation that has vertical and horizontal components [1]-[2] and square excitation [3]. In this study, we introduce a second harmonic component into the parametric excitation and examine its influence on the dynamics in the parametric pendulum. The second harmonic is derived from a force exerted by the rotating motion of the pendulum on the pivot point in an actual system. The dimensionless equation of motion for the parametric pendulum with the second harmonic excitation is described by

$$\ddot{\theta} + \gamma\dot{\theta} + [1 + p\cos\omega t + \alpha\cos(2\omega t + \phi)]\sin\theta = 0, \tag{1}$$

where γ denotes the damping coefficient, the term $p\cos\omega t$ the conventional parametric excitation with the amplitude p and the angular frequency ω, and $\alpha\cos(2\omega t+\phi)$ the second harmonic excitation introduced in this study. For the second harmonic, α and ϕ stand for the amplitude restricted in $0<\alpha<1/3$ and the phase difference from the fundamental.

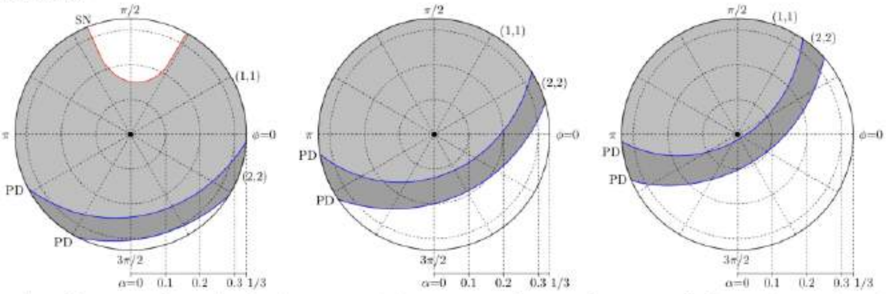

Figure 1: Bifurcation diagrams of rotation in the system (1) at γ=0.1 and ω=1.5 for p=0.4 (left), 0.5 (center), and 0.6 (right).

We clarify the existence domain of periodic rotations in the excitation parameter plane by using bifurcation diagrams shown in Fig. 1. Each diagram is expressed in polar coordinate of (α,ϕ) and its center point indicates a condition without a second harmonic. The light gray corresponds to the existence domain of rotation at which the pendulum rotates once during the excitation period and the dark gray that of rotation generating at the period doubling bifurcation from the rotation for the light gray. The second harmonic excitation with large amplitude disturbs the rotating motion for a certain range of the phase difference near $\phi=\pi/2$ and $\phi=5\pi/3$. The disturbance is caused through the saddle-node bifurcation at near $\phi=\pi/2$ and the period doubling bifurcation at near $\phi=5\pi/3$. Thus the dynamics associated with periodic rotation is non-uniformly disturbed for the phase difference ϕ. The right figure obtained at p=0.6 suggests that the second harmonic with small amplitude can disturb the rotating motion according to the excitation amplitude p. Since the domain corresponds to the condition of the parametric excitation for motion converting energy from nature, the understanding for the influence of the second harmonic is significant for the application

Acknowledgements

The author (YY) thanks Professor Marian Wiercigroch of the University of Aberdeen for research visit at the Centre for Applied Dynamics Research. He would like to express his sincere gratitude to Professor Takashi Hikihara of Kyoto University for fruitful discussion and continuous support with the experimental system. This research was partially supported by the Global COE program of Kyoto University and the Grant-in-Aid for Research Activity Start-up (23860039) to YY from the Ministry of Education, Culture, Sports and Technology of Japan.

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A NEW NORMAL FORM METHOD FOR ANALYSING SYSTEMS OF COUPLED NONLINEAR OSCILLATORS

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ABSTRACT

The method of normal forms has a long history, and involves applying transformations to the governing equations of motion with the aim of finding a simplified form. Of particular interest is the application of normal forms to find periodic steady-state system response solutions, as considered by Jezequel and Lamarque [1]. Potential advantages of using normal forms over other perturbation techniques for this type of problem include the ease in which it can be extended to consider multi-degree-of-freedom systems and non-autonomous systems and its suitability for analysis using symbolic manipulation programs.

In this paper we discuss a normal form technique that can be used to analyse systems of coupled nonlinear oscillators [2]. The method works by transforming the system into a simpler analytical form, from which both linear and nonlinear resonances are revealed. For each resonance the backbone curves and higher harmonic components of the response can be characterised [3]. The underlying mathematical technique is based on a near identity normal form transformation, which has an associated Lie group transformation. This approach is novel because the method works for systems of equations written in second-order form. This is a natural format for many dynamical applications, where the governing equations of motion are written in second-order form as standard practice. An example system, shown in Figure 1, which has cubic nonlinearities is used to show how the transformation can be recast to give a time independent representation of the system response.

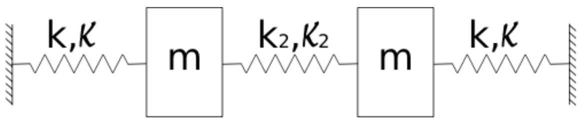

Figure 1: Schematic of the two degree-of-freedom nonlinear oscillator with cubic spring nonlinearities.

We also discuss how the analysis can be carried out with applied forcing, and how the approximations about response frequencies affect the accuracy of the technique. Example responses for the forced damped systems are shown in Figure 2. Here the dots are numerically computed, solid for increasing frequency and open for decreasing frequency. The solid lines are backbone curves that have been computed using normal forms.

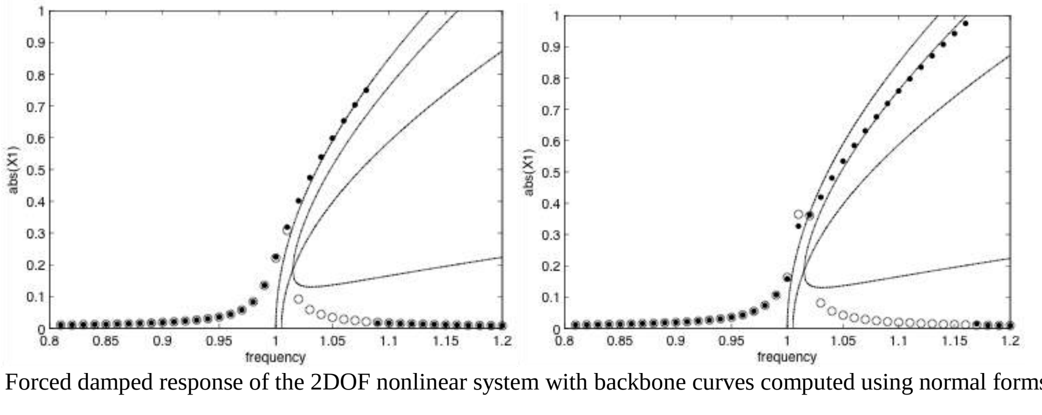

Figure 2: Forced damped response of the 2DOF nonlinear system with backbone curves computed using normal forms.

Finally in this paper, approximations to the nonlinear frequency response functions (FRFs) are obtained for the example system.

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Day 3: 23 August 2013

European Session (8:00 – 10:20)

“Synchronous states of slowly rotating pendula”, T Kapitaniak

“Non-linear viscous damping of floating bodies”, J M R Graham, M J Downie, M Hajiarab

“Localized modes in one-dimensional continuous structures of finite and infinite length”, D Indeitsev, Yu Mochalova

“Molecular flow engineering”, J M Reese, M Borg, N Dongari, K Ritos

“Effect of mesh size and spatial discretisation schemes on the solution of turbulent flow equations in agitated vessels”, D Dionisi

“Transitional wall-bounded flow: where do we stand?”, P Manneville

American Session (10:40 – 12:40)

“The rod dynamics under longitudinal impact”, N F Morozov, P E Tovstik

“Nonlinear vibration of a microbeam modeled by the strain gradient elasticity with an electrical actuation approximated by the Padé-Chebyshev method”, P Belardinelli, M Brocchini, L Demeio, S Lenci

“Rotor/stator interaction within a helicopter engine”, M Meingast, A Batailly, M Legrand, C Pierre

“Periodic behaviours in a 3D nonlinear system”, J Wang, C Liu

“Self-similarities of periodic structures for autoparametric oscillators”, S L T de Souza

“Bifurcations in frictional model of cutting process”, R Rusinek, K Kecik, J Warminski

Asian Poster Session (12:40 – 14:00)

“Impacts with friction”, S J Burns, P T Piiroinen

“Nonlinear dynamics of a vibro-impact machine subjected to electromagnetic interactions”, S C Jong, K C Woo, A A Popov

“Analysis and control of an underactuated drill-string”, Y Liu

“Impact-induced transients close to grazing in an impact oscillator”, D Manik, S Banerjee

“Stability analysis of a two pendulum system”, R A Morrison, M Wiercigroch

“Rotational motion of a parametric pendulum excited on a plane”, A Najdecka , M Wiercigroch,

“An approach to calibration of low dimensional VIV models using CFD”, A Postnikov, E Pavlovskaia, M Wiercigroch

“Characterisation of friction force and nature of bifurcation from experiments on a single-degree-of-freedom friction-induced system, A Saha, B Bhattacharya, P Wahi

"Attractor reconstruction for parameter identification in an impact oscillator", M Sayah, M Batista, J Ing, M Wiercigroch

"Experimental study of a drill-string assembly", V Vaziri, K Nandakumar, J Paez Chavez, M Wiercigroch

"Experimental study of control methods for maintaining rotation of parametric pendulum", V Vaziri, A Najdecka, M Wiercigroch

"Dynamic model and analysis of non-harmonic vibration of conveyor", Z Y Qin, Z L Zhao

"Bifurcation and quench control of grinding chatter", Y Yan, J Xu, M Wiercigroch

"Dynamics of compound bursting composed of different bursts and subthreshold oscillation", Z Q Yang, X Zhang

"A nonlinear model of balancing human standing", Q Xu, Z Wang

"Design of the delayed optimal feedback control for linear systems with multiple delayed inputs", Y Zhou, Z Wang

Russian Session (14:00 – 16:00)

"Periodic bifurcation analysis of fractional derivative and delay systems", A Y T Leung

"Exact solutions for discrete breathers in forced - damped chain", O V Gendelman

"Stability analysis for intermittent control of co-existing attractors", Y Liu, J Ing, M Wiercigroch, E E Pavlovskaia

"Crisis in chaotic pendulum with fuzzy uncertainty", L Hong, J-Q Sun

"Forced nonlinear normal modes in one disk rotor dynamics", N Perepelkin, Yu Mikhlin

"Constructing simple chaotic systems with an arbitrary number of equilibria or of scrolls", G Chen, X Wang, S Yu

Polish Session (16:20 – 18:20)

"Forward and backward motion control of a vibro-impact capsule system", Y Liu, E Pavlovskaia, M Wiercigroch

"Monitoring of the characteristic parameter changes of a nonlinear oscillator by nonlinear system modelling and analysis", R S Bayma, Z Q Lang, Y Liu, M Wiercigroch

"How common periodic stable behavior appears in nonlinear dissipative (mechanical) systems", M S Baptista, E Medeiros, I L Caldas

"One-dimensional chaos in a system with dry friction", S Kryzhevich, N Begun

"Predicting the approach to an oscillatory instability at a Hopf bifurcation", J M T Thompson, J Sieber

SYNCHRONOUS STATES OF SLOWLY ROTATING PENDULA

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ABSTRACT

Coupled systems that contain rotating elements are typical in physical, biological and engineering applications and for years have been the subject of intensive studies. One problem of scientific interest, which among others occurs in such systems is the phenomenon of synchronization of different rotating parts. Despite different initial conditions, after a sufficiently long transient, the rotating parts move in the same way-complete synchronization, or a permanent constant shift is established between their displacements, i.e., the angles of rotation-phase synchronization. Synchronization occurs due to dependence of the periods of rotating elements motion and the displacement of the base on which these elements are mounted.

Recently, the rotational motions of the pendulum attracted more interest due to the concept of extracting energy from sea waves using pendulum dynamics proposed by Wiercigroch. This interest motivated a number of researchers to pose the question; under which condition can slowly rotating pendula synchronize. In this paper we plan to review the studies on the synchronization of rotating pendula and compare them with the results obtained for oscillating pendula.

We consider the dynamics of the system consisting of *n* pendula mounted on the movable beam. The pendula are excited by the external torques which are inversely proportional to the angular velocities of the pendula. As the result of such excitation each pendulum rotates around its axis of rotation. It has been assumed that all pendula rotate in the same direction or in the opposite directions. We consider the case of slowly rotating pendula and estimate the influence of the gravity on their motion.

For the pendula rotating in the same direction it can be shown that both complete and phase synchronizations of the rotating pendula are possible. We derive the approximate analytical conditions for both types of synchronizations and equations which allow the estimation of the phase differences between the pendula. Contrary to the case of oscillatory pendulums phase synchronization is not limited to three and five clusters configurations. For the pendula rotating in the opposite directions despite opposite directions of rotation different types of synchronization occur.

We classify the synchronous states of the identical pendula and observe how the parameters mismatch can influence them. We give evidence that synchronous states are robust as they exist in the wide range of system parameters and can be observed in a simple experiments.

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NON-LINEAR VISCOUS DAMPING OF FLOATING BODIES

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ABSTRACT

Certain modes of response of floating bodies in sea waves can show a highly resonant behaviour due to the low values of the corresponding wave radiation damping. Roll and sway responses of moored ship and FPSO hulls in beam waves are particular cases of this where the peak amplitudes are largely controlled by non-linear effects. The most important of these under most wave conditions is the degree of viscous damping generated by vortex shedding from the bilges of the hull [1]. This damping is determined by the degree of rounding of the bilge and the presence of devices such as bilge keels to enhance the damping. A computationally efficient method of calculating the damping under regular wave conditions by matching locally around the bilges 2-D sectional time-stepped computations of separated flow and vortex shedding to the outer potential flow resulting from the incident-wave – body interaction, was first developed for single frequencies and 2-D hull shapes of rectangular cross-section in [2]. The present paper describes the further developments of the technique which match the 'inner' vortex shedding to more general 3-D 'outer' wave – body interactions and for more general hull shapes. Classical boundary-integral (panel) methods are used to compute the outer potential flow in the frequency domain for the efficient handling of large numbers of wave frequency and amplitude cases. Transfer of boundary condition information between the frequency domain of the outer wave potential flow field to the time domain of the inner separated flow field is carried out in the matching process. Computed damping and response results are compared with measurements from laboratory experiments for model scale ocean transport barges in regular waves showing very good prediction of roll RAOs, as in Figure 1 below, without the need for any empirical damping coefficients. As shown in figure 2 the viscous roll damping is quadratic when the hull sides are vertical and the bottom horizontal as they join the bilge and approximately quadratic for other section geometries. It is also apparent (particularly strongly in the case of sway as shown in figure 3) that while the non-linear additional effect of the vortex shedding leads at most frequencies to reduction in amplitude of response there is a frequency range where the vortex shedding, because of its phasing relative to the body motion, is able to generate a significant excitation of the response. This has also been observed in physical experiments. The paper will discuss these non-linear effects and also show the predicted effects of bilge rounding and bilge keels. Finally, a method will be presented of treating this non-linear damping under random excitation conditions to predict responses of general floating bodies in seas containing a broad-band spectrum of waves.

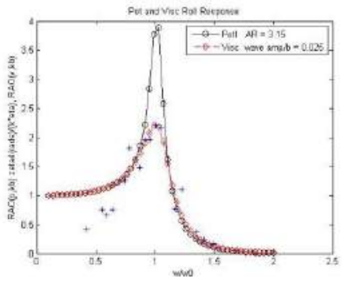

Figure 1: Roll RAO in regular waves
Black: potential, Red: total,
+ experimental measurements.

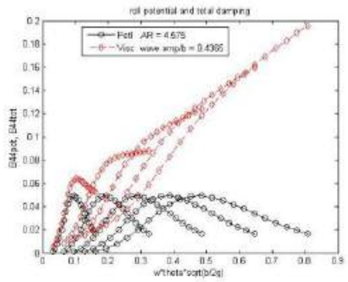

Figure 2: Forced Roll Damping Coefficient
Black: potential, Red: total
Θ = roll angle (0.087 – 0.44 rads).

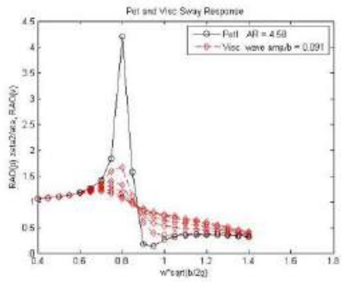

Figure 3: Sway RAO in regular waves
Black: potential, Red: total
Uniform increments in wave ht.

w = frequency (rads), b = hull beam, AR (hull section aspect ratio) = beam/(2 x draught).

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LOCALIZED MODES IN ONE-DIMENSIONAL CONTINUOUS STRUCTURES OF FINITE AND INFINITE LENGTH

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ABSTRACT

Free oscillations of continuous structures (strings and beams) on non-homogeneous elastic foundation are analyzed. Of interest is the existence of the localized modes, which is equivalent mathematically to the existence of discrete eigenfrequencies (possibly, embedded into the continuous spectrum) [1]. It is known that defects of the foundation can localize the first eigenmodes in a restricted region of the finite-length structures [2]. On the other hand, for the structures of infinite length localized modes exist and the solutions are localized near defects. Analogies between the two problems are shown. Conditions under which the discrete spectrum of the infinite-length structures coincides with the values of natural frequencies for the finite-length structures are defined. The problem reduces to an integral equation analysis and study of the behavior of the natural frequency as a function of the length of the structures. The effect of boundary conditions and positions of defect are analyzed.

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MOLECULAR FLOW ENGINEERING

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ABSTRACT

Molecular dynamics (MD) can be a sophisticated numerical tool for unpicking the behavior of systems at the smallest scales [1-7]. It simulates atomistic or molecular interactions and movements directly through Newton's laws and realistic intermolecular potentials [7]. Previously employed mainly for analyzing biological and chemical processes at equilibrium, MD is being used increasingly in the mechanical and aerospace engineering communities to shed light on non-equilibrium fluid systems, in particular, micro and nano flows. For these types of flows, without a molecular-level understanding, conventional simulation methods fail to provide accurate results. For example, a number of unexpected physical phenomena in nano flow technologies arise due to size and surface effects [1,3,5,6]. Molecular dynamics is also a fundamental component of many multiscale or hybrid fluid dynamics techniques that attempt to link the micro flow physics to the bulk system behavior [2,4].

This talk describes numerical experiments with molecular dynamics in the author's research group, and focusses on important new phenomena these experiments have uncovered that have not been accessible in other ways before. The hardware and software challenges of accelerating the computation of molecular interactions in systems of millions of interacting molecules is described; the advantages and limitations of molecular dynamics for practical engineering simulations are highlighted, and examples given.

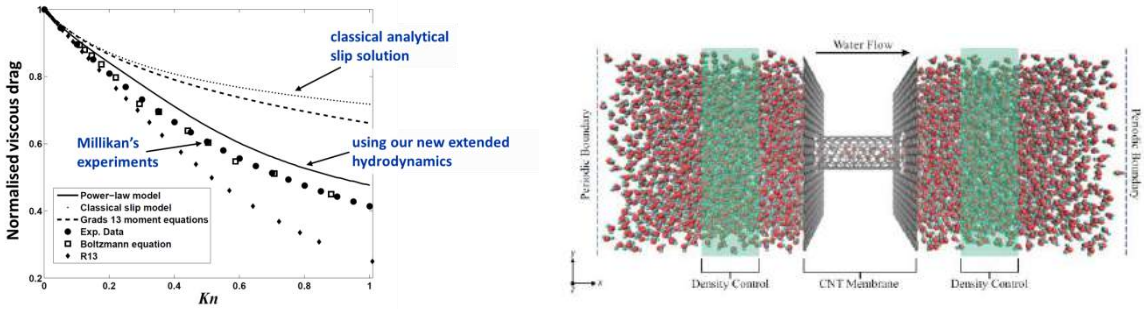

Figure 1: Molecular dynamics simulations give insight into (left) extended hydrodynamic models for the viscous drag of spheres in rarefied gases, and (right) the phenomenal transport rates of water through carbon nanotube filtration membranes.

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EFFECT OF MESH SIZE AND SPATIAL DISCRETISATION SCHEMES ON THE SOLUTION OF TURBULENT FLOW EQUATIONS IN AGITATED VESSELS

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ABSTRACT

Agitated vessels are commonly used in the chemical industry for many different purposes, including e.g. single or multiphase reactions, solid suspension, formulation, crystallization, etc. The use of Computational Fluid Dynamics (CFD) is gaining increasing interest for the simulation of agitated vessels with various aims, e.g. in order to simulate scale-up/scale-down, to optimize mixing, to select the optimum agitator.

In spite of the increasing use of CFD in the scientific literature, the numerical solution of CFD equations still poses significant challenges. In particular, simulation of turbulent flow is a matter of debate.[1] In most CFD simulations of agitated vessels reported in the literature, turbulence is simulated with the "k-ε" model,[2] based on the turbulent kinetic energy (k) and on the turbulent energy dissipation rate (ε). However, inaccuracies in the turbulent flow simulations with the k-ε model are often reported, and the model is considered to give an underestimation of the turbulent energy.[3]

The present work is aimed to study the numerical solution of CFD equations, in order to investigate the role of numerical accuracy in the reported poor prediction of turbulence profile with the k-ε model.

A 1-l agitated vessel with a radial-flow turbine and standard geometry was used in the simulations.[4] The mesh size was varied in the range 3.3E3 to 3.2E6 cells, i.e. up to a number of cells larger than in the majority of published CFD simulations of agitated vessels. For each mesh size five different spatial discretisation schemes for momentum, k and ε were investigated: First Order Upwind (FOU), Second Order Upwind (SOU), Power Law, Quadratic Upwind Interpolation for Convective Kinetics (QUICK) and Third Order Monotone Upstream-Centered Scheme for Conservation Laws (Third-Order MUSCL). The effect of the mesh size and of the discretisation scheme was evaluated on the following calculated parameters: k and ε spatial profiles, calculated at 5 different locations in the vessel, and agitator power draw, calculated both as overall moment on the agitator and shaft around the rotational axis and as integral of ε over the vessel volume.

The results can be summarized as follows:

- Both the mesh size and the discretisation methods have a strong influence on k and ε spatial profile and on the power draw calculated as integral of ε. On the other hand, power draw calculated as overall moment on the agitator and shaft is approximately constant (within 5% difference) for any mesh size and discretisation schemes in the investigated range.
- For the same mesh size, higher order schemes (SOU, QUICK, Third-Order MUSCL) give higher values of k and ε, and of the power draw calculated as integral of ε. E.g. with a mesh size of 1.1E6 cells, the calculated value of k in a location close to the impeller is almost 10 times higher with the Third-Order MUSCL scheme than with the FOU one.
- Mesh independence for the calculation of k and ε can be obtained only with very large mesh sizes, higher than 2E6 cells, i.e. larger than in most published studies on CFD applied to agitated vessels.
- Contrary to what often reported in the literature, the agitator power draw calculated as the overall moment on the agitator and shaft around the rotational axis and as integral of ε give virtually the same results (within 5% difference) provided that a very fine mesh (higher than 2E6 cells in this case) and a higher order discretisation scheme (SOU, QUICK or Third-Order MUSCL) are used.
- The obtained results indicate that the often reported inaccuracy of turbulence prediction with the k-ε model is likely to be due to numerical errors rather than to model inadequacy.

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TRANSITIONAL WALL-BOUNDED FLOW: WHERE DO WE STAND?

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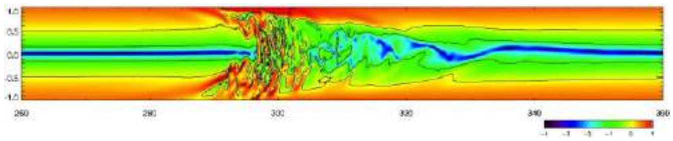
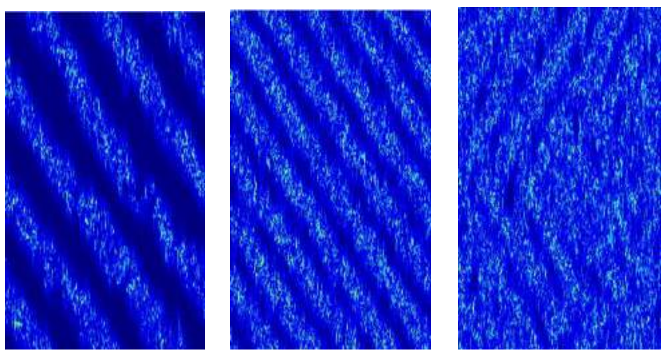
ABSTRACT

In fluid systems experiencing linear instability when le control parameter, e.g. the Reynolds number *Re*, is increased, the transition to turbulence is generally well understood in terms of a cascading process progressively complicating the dynamics from laminar to disorganized flow. Convection of a fluid heated from below in a closed container is a typical example helping us to apprehend the role of confinement by lateral boundaries. In open flows, given laminar velocity profiles can be stable or unstable against mechanisms of inertial origin (Kelvin–Helmholtz instability). Free shear flows with inflectional profiles are unstable at low *Re* and become turbulent in a progressive, globally supercritical, manner. Flows developing along solid boundaries, with non-inflectional base profiles, are inertially stable and may only become unstable against perverse effects of viscosity at very large *Re* (Tollmien–Schlichting waves) [1]. In some cases – flow in a pipe (Poiseuille), simple plane shear flow (Couette) – the viscous linear mechanism does not show up before *Re* reaches infinity. However, except in ideally prepared systems, such flows usually become turbulent at intermediate values of *Re*, under the effect of finite amplitude perturbations, either triggered or present in the residual turbulence (natural transition). We shall review results recently obtained in this context, focusing on the different approaches that have been followed to capture the nontrivial branch of flow regime away from laminar flow, low vs. high dimensional dynamical systems, chaos vs. spatiotemporal chaos, non-modal amplification of perturbation energy and by-pass transition, transient vs. sustained regimes. We shall specifically consider the determination of the global stability threshold Re_g in pipe flow, which can be understood as the value of *Re* when the expansion of turbulence by puff splitting overcome its decay due to the finite lifetime of puffs associated to transient chaos [2]. A similar situation holds for plane Couette flow except for spatial extension that is now two-dimensional in the plane of the flow instead of one-dimensional along the tube. In addition to Re_g this adds the possibility of a second, higher threshold, Re_t above which turbulence is essentially featureless, while between Re_g and Re_t is presents itself as an alternation of oblique laminar and turbulent bands [3]. Beyond the academic context and the paradigmatic cases of pipe Poiseuille flow and plane Couette flow, the general features of the direct (discontinuous) transition to turbulence [4] can be recovered in the general class of flows bounded by walls where the no-slip condition applies of great practical interest, e.g. standard (Blasius) boundary layer flows.



Puff in Poiseuille pipe flow, after [5].

From left to right: Band pattern close to Re_g, between Re_g and Re_t, and close to Re_t in plane Couette flow (wide domain), after [6].



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THE ROD DYNAMICS UNDER LONGITUDINAL IMPACT

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ABSTRACT

Interaction of the longitudinal and bending rod vibrations under action of a short longitudinal impact is investigated. First the elastic system behavior under action of the compressing dynamical force with the value which essentially exceed the Euler critical force [1]. It is established that the maximal growth of the bending amplitude has place for one of the buckling modes with high number. The close problems are investigated in [2]. In [1,2] the reason of lateral motion is connected with the initial imperfections.

As a rule, in problems of the rod stability under longitudinal dynamical loading it is supposed that the compressing force is constant with respect to the longitudinal co-ordinate. In [3] the short longitudinal impact on the rod end is studied. It is supposed that the impact time is shorter than the time during which the axial wave covers two times the rod length. The wave reflects from the clamped and free rod ends, and it leads to a periodic longitudinal compression and extension. Introduce dimensionless variables for which both the rod length and the wave velocity are equal to unit. Then the length of impact τ is less than 2 and the period is equal of longitudinal excitation is equal to 4. In Fig. 1 a semi-strip $0 \le x \le 1, 0 \le t < \infty$ which is divided in the parts with compression (+), zero (0), and extension (-) longitudinal stresses, is shown.

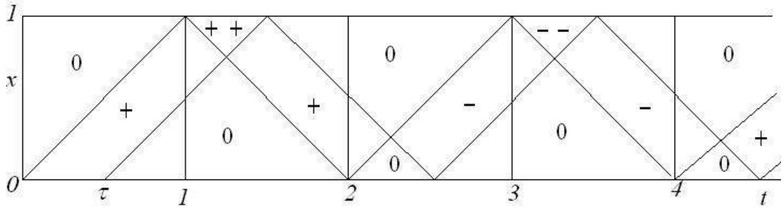

Fig. 1.

In [3] in linear approximation the bending rod vibrations in presence of the initial imperfections are studied, and the parametric resonance is noted. Here the parametric stability of bending vibrations is investigated in the case when the imperfections are absent. The problem is approximately reduced to the system of linear ordinary differential equations with periodic coefficients. The characteristic indices ρ are found. If $|\rho| > 1$ then the unbounded growth of amplitude has place. This result is incorrect because the studied mechanical system is conservative when impact is finished. That is why the nonlinear system in partial derivatives describing the axial-bending vibrations is obtained. By using the Bubnov-Galerkin method this system is reduced to the system of ordinary differential equations. The Cauchy problem for this system with the non-zero initial conditions is solved numerically. For some values of the system parameters the mutual transferring of axial and bending vibrations is noted. In Fig. 2 two examples of the bending amplitude are presented. This phenomenon is similar to the vibrations of a pendulum on the elastic suspension. Also it is interesting to take into consideration the visco-elastic forces.

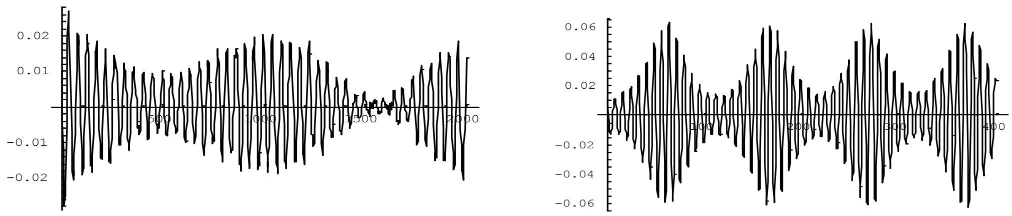

Fig .2.

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NONLINEAR VIBRATIONS OF A SLENDER MICROBEAM MODELED BY MEANS OF THE STRAIN-GRADIENT ELASTICITY THEORY AND WITH AN ELECTRIC ACTUATION APPROXIMATED BY THE CHEBYSHEV METHOD

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ABSTRACT

Control and design of Micro-electro-mechanical systems (MEMS) is, nowadays, a fundamental step in several engineering fields. The mechanical modeling of an electrically-actuated microbeams, based on the non-classical continuum theories, can catch size dependent phenomena at the microscale. Considering the strain-gradient elasticity theory, the strain energy density s, is a function both of the strain tensor ε_{ij} and of the second-order deformation gradient tensor η_{ijk}. In particular, according to [1], a modified first variation δs of the strain energy, written as:

$$\delta s = \sigma_{ij}\delta\varepsilon_{ij} + p_i\delta\gamma_i + \tau^{(1)}_{ijk}\delta\eta^{(1)}_{ijk} + m^s_{ij}\delta\chi^s_{ij}, \tag{1}$$

introduce three new material length parameters into the conjugate high-order static terms p_i, $\tau^{(1)}_{ijk}$, m^s_{ij}.

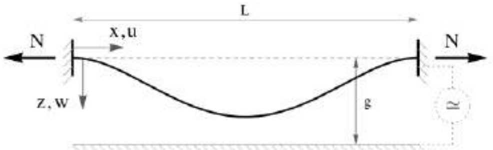

Figure 1: 1-D model for the electrically-actuated microbeam

Using the extended Hamilton's principle, we get the equations of motion for an electrically-actuated slender clamped-clamped microbeam (see Figure 1), under the assumption of the nonlinear Euler-Bernoulli theory and considering a second order axial stretch. The dimensionless problem, condensing the longitudinal equation of motion into the transversal one [2], reads:

$$\ddot{w}_d + c\dot{w}_d + w^{iv} - \alpha_3 w^{vi} = \left(N + \alpha_1 \int_0^1 w'^2 dx\right) w'' + \frac{\alpha_2 V^2}{(1-w)^2}, \qquad w = w' = w'' = 0 \quad \text{at} \quad x = 0, 1. \tag{2}$$

The previous equation takes into account, through N, an externally applied load, while the geometric nonlinearity due to the axis elongation is also considered by means of the integro-differential term. Neglecting all time dependences we get the steady state problem, solved numerically with the generalized differential quadrature method (GDQM). We found that an increasing value of α_3, the new parameter generated by the use of the non-classical approach and function of higher-order material parameters, staves off the risk of the instability in pull-in regime for the static problem.

For the dynamical study, a classic approach consists of using the Taylor series expansion for the nonlinear electric term, which is characterized by a very small error only around the center of the expansion [3]. The alternative applied in this work is represented by the use of the Chebyshev polynomials, spreading smoothly the error over the domain. Thus, the our formulation can be successfully applied in all prescribed domains with a limited and reduced approximation error. Investigating on the dynamic problem with a Galerkin procedure, we found a flexible nonlinear behaviour, with a competition between a softening and an hardening response. Evaluating the single-degree-of-freedom problem as function of different parameters, it can be noted that for low electric voltage V the behaviour changes, from soft to hard, increasing α_1, that is proportional to the square of the ratio between the initial gap g and the beam thickness.

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ROTOR/STATOR INTERACTION WITHIN A HELICOPTER ENGINE

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ABSTRACT

Modern helicopter engines feature small operating clearances particularly around centrifugal impellers such as the one depicted in Fig. 1. Under extreme operating conditions, these clearances may be fully consumed leading to direct structural contacts between the rotating assemblies and the respective surrounding casings. Induced by structural unilateral contacts, interaction phenomena have already been reported for axial compressors [1] and have been systematically investigated by manufacturers [2] due to their potentially dramatic consequences on the engines. For instance, recent investigations [2] and experimental observations [1] have shed some light and shown the relevance of the employed numerical strategy [3] for axial compressors. However, impellers are still challenging due to the inherent complexity of the blades geometry.

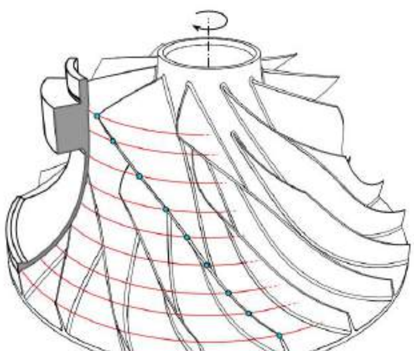

Figure 1: Impeller of interest

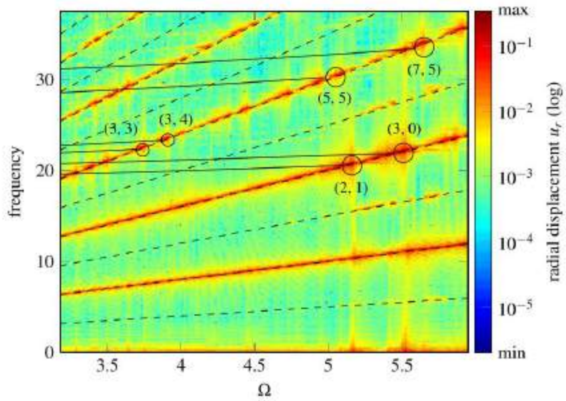

Figure 2: Interaction map

The present work aims at providing the additional tools for complementary analyses of impeller/casing unilateral contact investigations. Two-dimensional Fourier transforms of the results in time and space are carried out and interaction maps (as depicted in Fig. 2) are displayed over a wide rotational speed range are performed. A meticulous tracking of each harmonic allows for the identification of interactions involving a single blade. Exploration of the contact locations on the casing during the interaction also provides valuable information on the dynamics of the impeller.

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PERIODIC BEHAVIORS IN A 3D NONLINEAR SYSTEM

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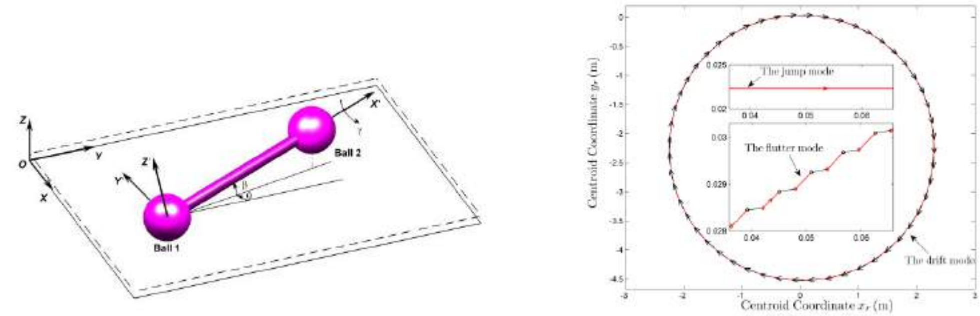
ABSTRACT

We studied a nonlinear system (see Fig. 1a), which is a 3D rigid body, termed as a dimer, composing of two identical solid spheres rigidly connected by a light-weight rod. By letting it bouncing upon a horizontal plate that vertically vibrates following a harmonic signal $z_p = A_z \sin(\omega t + \phi_0)$, previous experimental work [1] and numerical simulation [2] have investigated that the planar dynamics of the system can reveal several kinds of periodic modes due to the concatenations of a series of complex contact and impact states. In this paper, we will show that the common case for the 3D dynamics of the system contains more valuable results, in which the striking phenomena are presented in the 3D system.



(a) (b)

Figure 1: (a) The system of a dimer bouncing on plate with a harmonic vibration $z_p = A_z \sin(\omega t + \phi_0)$. (b) The trajectories of the mass centre of the dimer projected on a horizontal plane under drift, jump and flutter modes. In which arrows positioned in the orbits signify velocities of the mass centre at the corresponding instants, and different symbols correspond to different states.

Let (x_1, y_1, z_1), (x_r, y_r, z_r) be coordinates of the mass centre of the ball 1 and the dimer, respectively. The system takes six degrees of freedom with generalized coordinates defined by $\mathbf{q} = (x_1, y_1, z_1, \theta, \beta, \gamma)^T$. The governing equations of the bouncing dimer are

$$\boldsymbol{M}(\mathbf{q})\,\ddot{\mathbf{q}} - \boldsymbol{h}(\mathbf{q}, \dot{\mathbf{q}}, t) = \boldsymbol{Q} + \boldsymbol{W}\boldsymbol{F}^n - \boldsymbol{N}\boldsymbol{F}^\tau \qquad (1)$$

where $\boldsymbol{M}(\mathbf{q}) \in \mathbb{R}^{6\times 6}$ is the mass matrix, $\boldsymbol{h}(\mathbf{q}, \dot{\mathbf{q}}, t)$ and $\boldsymbol{Q}$ are respectively the vectors of the inertial forces and the generalized forces induced by gravity.

Based on a general theory for solving systems with frictional multiple contact/impacts[2], theoretical analyses and numerical simulations are performed accurately, and demonstrate that three nontrivial periodic motions, named as drift mode, jump mode, and flutter mode, can be triggered under different driving parameters relating the amplitude and the frequency of the harmonic oscillation. Actually, the normal motions of these periodic behaviours are nearly the same as the 2D case since the 3D motions can't change the state combinations confined in different modes. However, the couplings inherently resided in the rigid body dynamics between the translation and rotation extremely change the manifestation of the dynamics through the continual contact and impact interactions between the dimer and the rough plate. Therefore, nontrivial phenomena can be presented in the trajectories of the mass center of the dimer projected on a horizontal plane as shown in Fig. 1b, in which the drift mode corresponds to a closed circular orbit derived from a new periodic motion, the flutter mode is related with a dog-leg path, while the jump mode is relevant to a straight line.

Theoretical analysis indicates that the new periodic behaviour relating to a close circle orbit is formed owing to the symmetrical breaking of the phase orbits in the motions of the dimer under in a drift mode.

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SELF-SIMILARITIES OF PERIODIC STRUCTURES FOR AUTOPARAMETRIC OSCILLATORS

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ABSTRACT
We investigate the dynamics of an autoparametric Duffing system [1,2] in the two-dimensional parameter space by using Lyapunov exponents.

Figure 1 shows a schematic model of the autoparametric oscillator that is composed of a cart, with a pendulum, connected to a fixed frame by a nonlinear spring and a dash-pot. We denote by *x* the displacement of the cart and by φ the angular displacement of the pendulum.

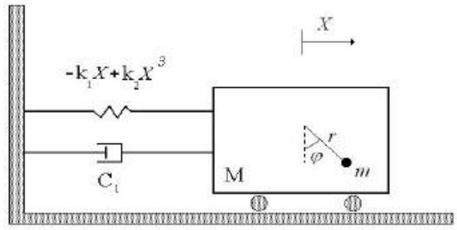

Figure 1: Schematic model of the autoparametric oscillator.

The equations of motion for both the cart and the pendulum are given by:

$$(m+M)\ddot{x}+c_1\dot{x}-k_1x+k_2x^3+mr(\ddot{\varphi}\cos\varphi-\dot{\varphi}^2\sin\varphi)=F\cos\omega t\,,$$
$$mr^2\ddot{\varphi}+c_2\dot{\varphi}+mgr\sin\varphi+mr\,\ddot{x}\,\cos\varphi=E_1.$$

In order to characterise the dynamics of the oscillator in two-dimensional parameter space, we construct several diagrams by using the Lyapunov exponents. On examining the diagrams, we identify a vast quantity of periodic structures formed by Arnold tongues [3]-[5] and shrimps [6,7]. (Here, numerical simulations of the system were performed by using the fourth-order Runge-Kutta method with a fixed step.)

In conclusion, we have identified self-similar periodic structures, such as Arnold tongues and shrimps. The period of the Arnold tongues is associated with a Fibonacci sequence [8]. In addition, we have observed period-adding sequences with accumulation horizons for both periodic structures.

Acknowledgments: This work was made possible by partial financial support from CNPq (Brazilian government agency).

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BIFURCATIONS IN FRICTIONAL MODEL OF CUTTING PROCESS

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ABSTRACT

Nowadays, a cutting process is still very popular method of manufacturing. The industries, specially aerospace and aeronautical, are looking for more effective ways of cutting which can avoid unwanted self-excited vibrations. This kind of vibrations, between the tool and workpiece called chatter, is a primary cause of surface finish roughness [1,2]. Therefore, in this work the idea of chatter suppression with the help of semi-active chatter control (ACC) system [3] is studied. The system is based on piezoelectric elements which excite ultra-vibrations of the workpiece (Fig.1). Since, the dry friction phenomenon is one of the reasons of chatter, therefore we have decided to use ACC system to suppress self-excited vibrations in the nonlinear model of cutting. The chatter vibrations in our model are generated by the frictional effect between the tool and the workpiece.

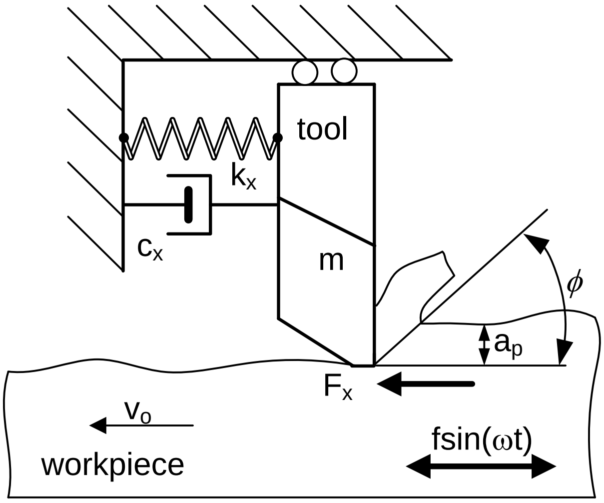

Fig1. Model of cutting process with ACC system

In this contribution, we would like to explain interaction between self-excitation mechanism modelled as a classical Rayleigh's term [4] and ultra-vibrations excited by the external force. Moreover, we expect the system exhibits interesting nonlinear behaviour leading to bifurcation of the solutions and other nonlinear behaviours. Therefore, the influence of the external excitation parameters on chatter vibrations will be tested. The model of cutting process with ACC system will be solved numerically and/or analytically and verified experimentally to confirm the ACC system can suppress chatter vibrations.

Acknowledgements
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IMPACTS WITH FRICTION

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ABSTRACT

Impacts between two objects can be modelled as rigid or compliant [1] depending on the materials of the impacting objects. Here we will focus on rigid bodies and common assumptions for rigid-body impacts are that they occur over infinitesimal time periods, they occur at discrete points and there is no compliance of the contact points. As a consequence of these assumptions, modelling impacts with friction can present many difficulties. A well-known example is that of the *Painlevé paradox* [2]. The Painlevé paradox is the phenomenon whereby a rigid body sliding along a surface can experience normal velocity jumps, i.e. there is a jump in the normal acceleration that can be seen as an impact without collision [3].

Here we derive an impact mapping for planar rigid-bodies using the *Amontons-Coulomb law* and *Newton's restitution law*, following the approach of Brach [4], and compare it with a recent impact mapping derived by [3] and [1]. The notion of the *impulse ratio* μ is introduced and analysed, and the conditions for which the impulse ratio is equivalent to the conventional coefficient of friction are discussed. In particular we will show the need to impose two bounds on the impulse ratio. First, μ_{cr} is the value of μ for which the tangential contact point velocity goes to zero after impact. Second, μ_E is the value for which there is no kinetic energy loss at impact.

As a specific example consider the case of a slender rod impacting against a non-compliant plane. In Figures 1(a) and 1(b) we plot μ_{cr} and μ_E when varying the tangential and angular velocities V_T and dθ/dt, respectively, for two different values of the coefficient of restitution *e*. The figures highlight how to choose a value for μ such that non-physical impacts can be avoided.

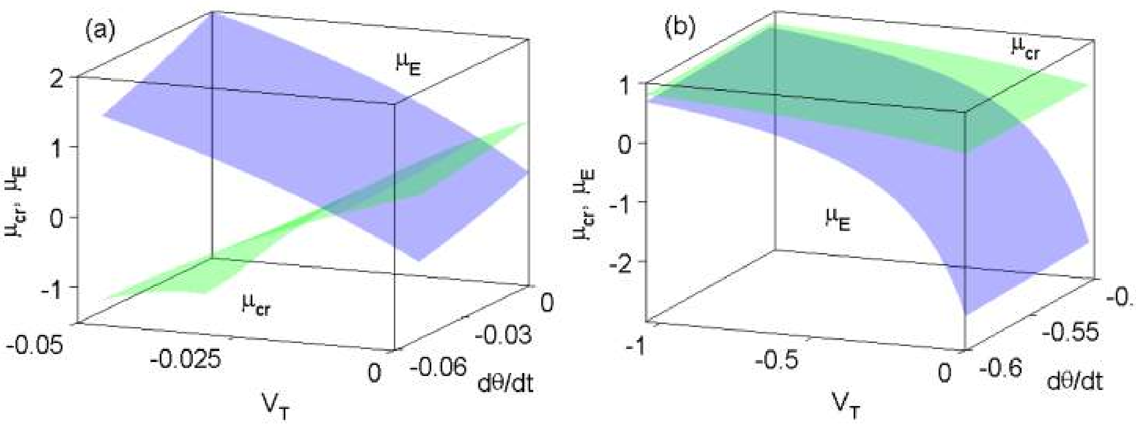

Figure 1 Plots of μ_{cr} (green) and μ_E (blue) for a slender rod with mass m = 1, length L = 2 and radius of gyration r = 0.33. In (a) the impact angle θ = 2.9, impact normal velocity v = -0.1 and restitution coefficient e = 0.1, and in (b) θ = 1.9, v = -1.1 and e = 1.

This work can give informative predictions for practitioners and researchers working on and modelling impacts with friction. The analysis will also give an insight under what parameter values and geometric configurations the derived impact mapping will have a physical meaning.

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NONLINEAR DYNAMICS OF A VIBRO-IMPACT MACHINE SUBJECTED TO ELECTROMAGNETIC INTERACTIONS

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ABSTRACT

The previous vibro-impact machine developed by Nguyen [1] is able to produce a forward progression in horizontal motion for ground moling purpose with a bulky frame as support. This paper describes a new vibro-impact machine designed and manufactured (see Figure 1a), being more compact in geometry and able to perform in vertical orientation for more versatile applications. Other than ground moling, it is also tested to be able to crush rocks and bricks sample. The penetration through a hard and brittle material is done with a vertical downward progression induced by impact force within the machine. The experimental rig consists of two solenoids, connected to a direct current (DC) and alternative current (AC) power supply each, generating attraction force at opposite direction onto a mild steel cylindrical conductor bar that oscillates within the solenoids. As the oscillating conductor bar comes into contact with a stopper, the inertia of conductor bar creates an impact on the rig and then is transmitted to the brick/rock sample (see Figure 1b).

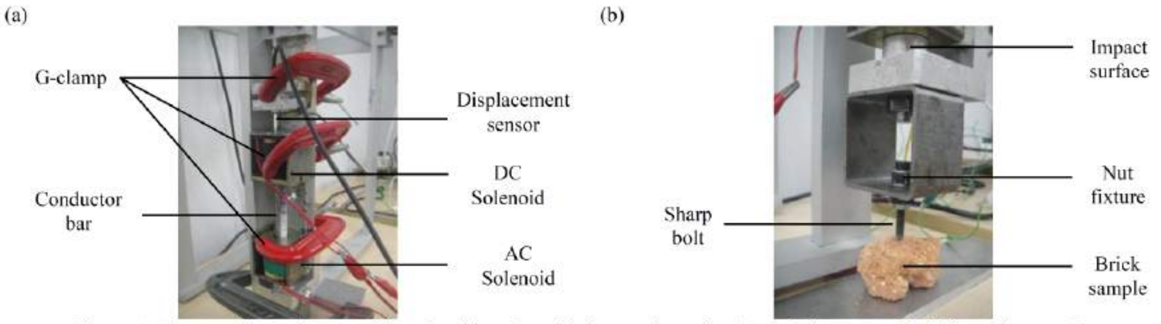

Figure 1: The experimental setup of the solenoids and oscillating conductor bar (a), and the setup of brick breaking test (b).

The variation of the operating frequency of switching the AC power supply between on/off is the main system parameter of interest, as it greatly affects the conductor bar's oscillation amplitude and response frequency. System parameters and power supply are manipulated in National Instruments hardware and LabVIEW software, which the input signals are generated from a computer. Experimental data are presented in time histories, which the oscillations of conductor bar and the downward progression of experimental rig into the tested material are recorded using a Linear Variable Differential Transformer (LVDT) sensor. A discontinuous mathematical model of the oscillation of conductor bar has been derived. Equation (1) describes the dynamics of the conductor bar in free oscillating region:

$$m\ddot{x} = F_1 - F_2 - mg, \quad \text{at } x > 0 \qquad (1)$$

where m is the mass of the conductor bar, F_1 is the electromagnetic force of DC solenoid, F_2 is the electromagnetic force of AC solenoid and g being the gravitational acceleration. The displacement of conductor from the impact surface is denoted as x. Equation (2) describes the dynamics of the conductor bar during impact:

$$m\ddot{x} + c\dot{x} + kx = F_1 - F_2 - mg, \quad \text{at } x \leq 0 \qquad (2)$$

where c and k are the damping coefficient and stiffness of the impact surface, respectively. The solution of numerical integration of the mathematical model is compared with the experimental data to validate its ability to predict the conductor's dynamics behaviour. Similarity in overall waveform is obtained. The model will be used in order to optimise the machine design.

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ANALYSIS AND CONTROL OF AN UNDERACTUATED DRILL-STRING

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ABSTRACT

Oil well drill-strings exhibit complex dynamical phenomena which includes undesired oscillations due to friction. Stick-slip, bit bouncing, and whirl motions are three main harmful oscillations that must be suppressed during a drilling operation. Indeed, stick-slip behavior exist in the 50% of drilling time [1], and the whipping and high speed rotations of the bit in the slip phase may cause both severe bit bouncing phenomena and whirl motion at the bottom-hole assembly (BHA) [2].

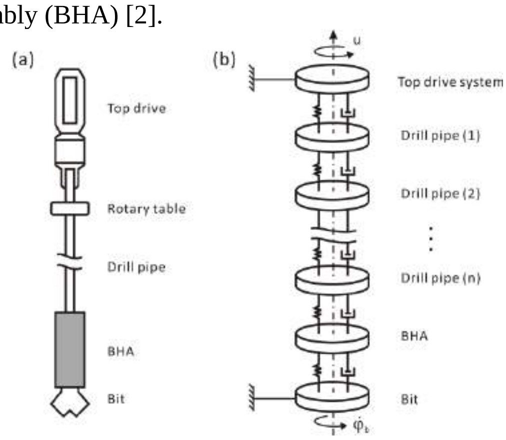

Figure 1: Schematics of (a) a drill-string system and (b) a simplified drill-string model.

This paper studies the stick-slip motion of a drill-string shown in Fig. 1(a) by using an underactuated drill-string model (see Fig. 1(b)), which can be written as

$$J\ddot{\Phi} + C\dot{\Phi} + K\Phi + T = U \tag{1}$$

where $\Phi \in \mathfrak{R}^{(n+3)\times 1}$ is the system state, $J \in \mathfrak{R}^{(n+3)\times(n+3)}$ is the inertia matrix, $C \in \mathfrak{R}^{(n+3)\times(n+3)}$ is the torsional damping matrix, $K \in \mathfrak{R}^{(n+3)\times(n+3)}$ is the torsional stiffness matrix, $T \in \mathfrak{R}^{(n+3)\times 1}$ is the friction torque, and $U \in \mathfrak{R}^{(n+3)\times 1}$ is the control torque input from the top drive system. The system in underactuated [3] as it has only one control input but n+3 degrees of freedom to be controlled. The control issue concerned here is to suppress the stick-slip motion of the bit, and makes the rotary speed track the desired speed by controlling the torque input with the existence of random time delays from sensor to controller and from controller to actuator as shown in Fig. 2(a). Model predictive control (see Fig. 2(b)) is used to address the control delay issue in this paper. Extensive simulation results will be given to demonstrate that the control method is effective.

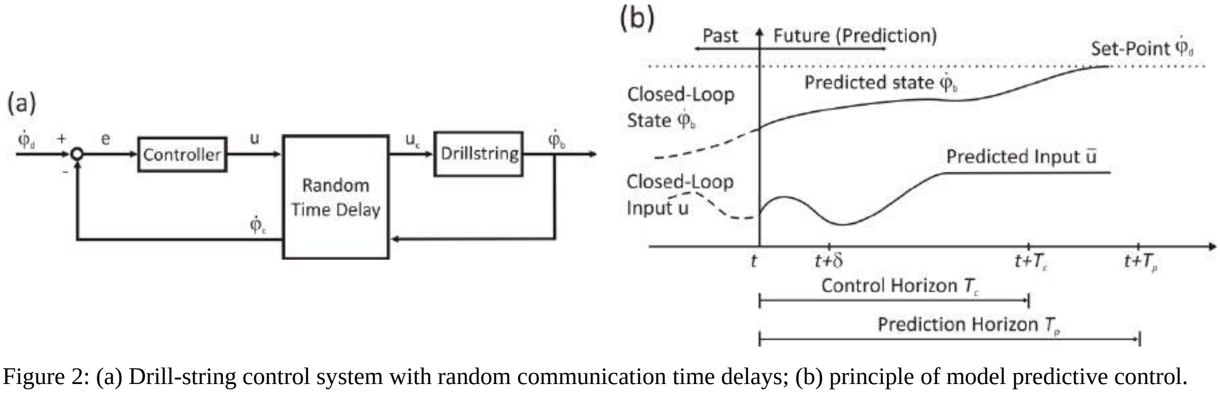

Figure 2: (a) Drill-string control system with random communication time delays; (b) principle of model predictive control.

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IMPACT-INDUCED TRANSIENTS CLOSE TO GRAZING IN AN IMPACT OSCILLATOR

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ABSTRACT

Through our earlier work [1,2,3] we found that impact oscillators exhibit long transients close to grazing, and it takes a very long time to reach a stable periodic behaviour. In the present work we investigate this phenomenon in detail using the simple mass-spring-damper impacting system shown in Fig.1.

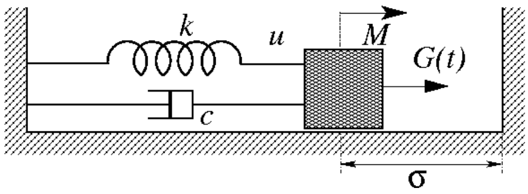

Fig.1: The system under consideration

We find that, as the parameter (say the strength of the forcing function or the damping coefficient) approaches the critical value corresponding to grazing, the transient time grossly follows a power law characteristics: $\tau \sim (\alpha - \alpha_*)^{-\gamma}$ in accordance with the findings of [4] for discrete time systems. This behaviour makes it extremely difficult to study the steady state behaviour of such systems in an experimental setup. We attempt to explain this power law behaviour by defining μ_{xv}: a closed neighbourhood in the phase space where, if the system once ends up, it does not undergo any future collision with the wall. Assuming that the behaviour of the system is ergodic so long as the transient is significant, we claim that μ_{xv} is inversely proportional to the lifetime of the transients in the system. In an attempt to justify our claim, we derive approximate analytical expressions for μ_{xv} for different regimes of validity and show that it also follows a power law characteristics, with an exponent that is opposite in sign to the exponent of transient lifetimes.

We also study the effects of transition from a hard wall to a soft wall on the transients, as well as that of varying the difference between the natural frequency and the forcing function frequency, $\omega_g - \omega$.

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STABILITY ANALYSIS OF A TWO PENDULUM SYSTEM

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ABSTRACT

Dynamics of pendulums and multiple pendulum systems with a view to develop energy extraction from sea waves have been studied in the Centre for Applied Dynamics Research at Aberdeen for several years [1]. The central idea is to utilize the rotating solutions as they are much easier to control and are more efficient than the oscillatory ones.

The initial investigations were focussed on the dynamics of a parametrically excited pendulum (see Figure 1(a), where the oscillatory and rotary solutions were determined both theoretically and experimentally [2-4]. Recent work by Lenci *et al.* [5] building on the earlier development by Xu and Wiercigroch [2] has developed a comprehensive range of analytical expressions which describe the rotating solutions of the parametrically excited pendulum. Starting with the exact solutions available for the Hamiltonian system the perturbation theory is developed to provide approximate expressions for the rotating motion. The analysis provides stability information for these solutions and confirms and extends the understanding of the bifurcation behaviour of the system.

There are number of challenges when designing practical systems to operate in an ocean environment. Systems must be physically robust to withstand long periods of operation without maintenance and also demonstrate robust stable solutions which can be maintained over a wide range of operating parameters. The consideration of a two pendulum system offers advantages in both these criteria. Firstly, there are a range of solutions, rotation in anti-phase, which have naturally balancing effects in terms of the forces exerted on the base point by the pendulum - meaning a practically operating system would be less prone to component fatigue. Secondly the naturally observed tendency for pendulums, even with very small coupling, to entrain into synchronised motions provides robust solution regimes in the parameter space, which can be exploited when developing stability theory and control.

This paper provides a stability analysis of such a two pendulum system, a schematic of which is provided in Figure 1.

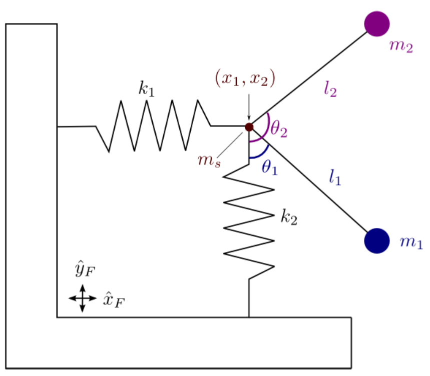

Figure 1: Schematic diagram of a two pendulum system

The non-dimensionalised equations of motion for the system, which were first recorded in [10], are

$$\ddot{\theta}_1 + \beta_p\dot{\theta}_1 + \sin\theta_1 = -\left(\ddot{x}_1\cos\theta_1 + \ddot{x}_2\sin\theta_1\right)$$
$$\ddot{\theta}_2 + \beta_p\dot{\theta}_2 + \sin\theta_2 = -\left(\ddot{x}_1\cos\theta_2 + \ddot{x}_2\sin\theta_2\right)$$
$$\ddot{x}_1 + \beta_1\left(\dot{x}_1 - \dot{x}_F\right) + \kappa_1\left(x_1 - x_F\right) = -M_r\left(\ddot{\theta}_1\cos\theta_1 - \dot{\theta}_1^2\sin\theta_1 + \ddot{\theta}_2\cos\theta_2 - \dot{\theta}_2^2\sin\theta_2\right)$$
$$\ddot{x}_2 + \beta_2\left(\dot{x}_2 - \dot{y}_F\right) + \kappa_2\left(x_2 - y_F\right) = -M_r\left(\ddot{\theta}_1\sin\theta_1 + \dot{\theta}_1^2\cos\theta_1 + \ddot{\theta}_2\sin\theta_2 + \dot{\theta}_2^2\cos\theta_2\right).$$

We consider the system under sinusoidal heave excitation, that is $\hat{x}_F(t) = 0$ and $\hat{y}_F(t) = \gamma_y \sin\omega_y t$. The system has four fixed points corresponding to the combinations of upended and hanging equilibrium configurations of the two pendulums.

Considering the equations linearised about the stable fixed point corresponding to the hanging equilibrium points of the two pendulums under this heave excitation allows for numerical calculation of the Floquet multipliers for selected excitation parameters.

Figure 2(a) presents Floquet multipliers for two pendulum system linearised about the origin. Values above one are coloured ‘hot’ on a scale from yellow through to red and representing increasing linear instability as the multiplier increases beyond one. Colder colours - shades of blue - represent multiplier values between zero and one where the system exhibits linear stability.

(a) (b)

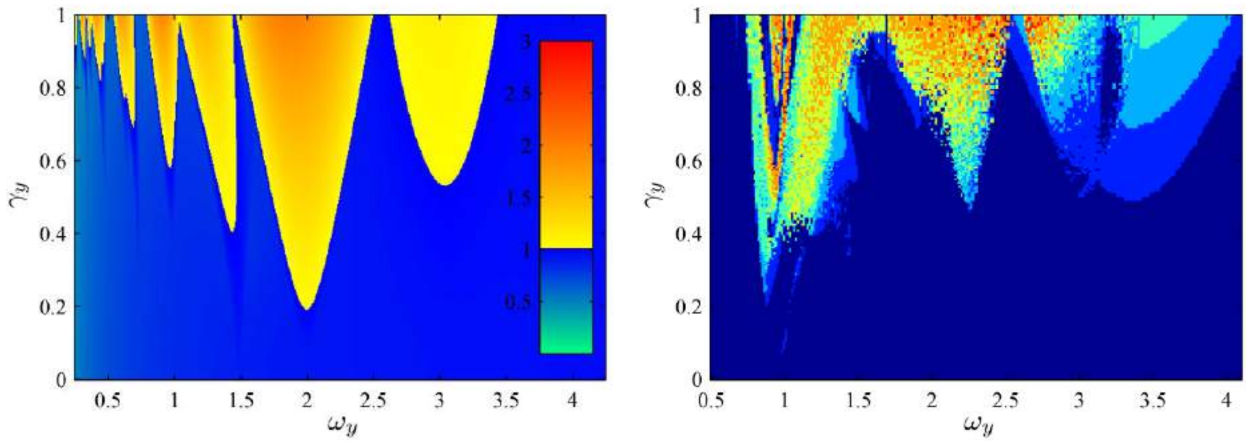

Figure 2: (a) Parameter stability plot (linear, Floquet), (b) parameter stability plot (nonlinear, Lyapunov)

Figure 2(b) presents the same parameter regime but with a Lyapunov parameter plot for the nonlinear system. In the Lyapunov plot the dark blue represents motions with Lyapunov exponents which are negative - i.e. non-chaotic motions lighter blue represent increasing values of the largest exponent with hotter colours representing motions which are chaotic.

The similarity in structure is clear and it is apparent that linear instability is a prerequisite for chaotic motion in the fully nonlinear system.

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ROTATIONAL MOTION OF A PARAMETRIC PENDULUM EXCITED ON A PLANE

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ABSTRACT

Rotational motion of a well known parametric pendulum could be potentially applied to hearvest the energy form the oscillations [1], provided that sufficient ampoint of energy is supplied to maintain rotational response. The aim of this paper is to demonstrate how modification of the forcing arrangement influneces the stability of the rotational motion and the minimum energy required to sustain rotation of a parametric pendulum. Dynamics of the pendulum excited along a tilted axes at an arbitrary angle α has been studied experimentally and numerically. Finally addtional rocking excitation on the pendulum base has been introducted and the study repeated. In both cases the minimum amplitude required for rotational respone has been compared with a classical parametric case demonstrating significant shift of the Saddle-Node bifurcation curve.

(a) (b) (c)

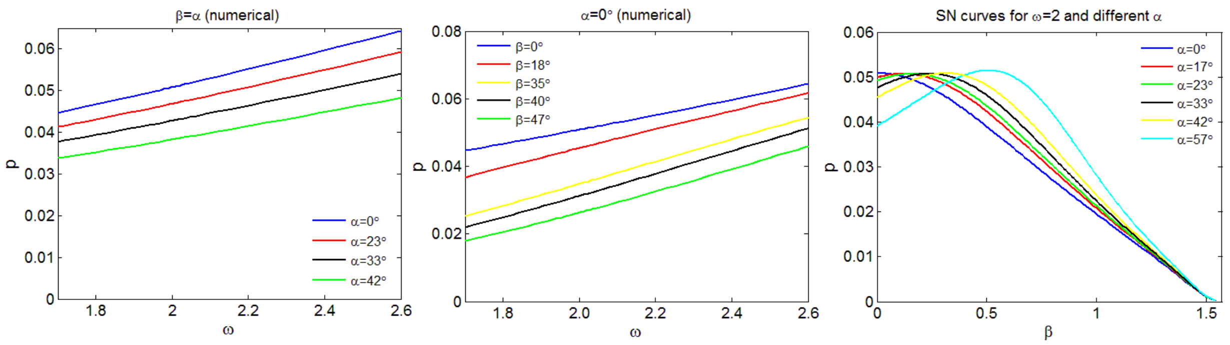

Figure 1: Numerically determined Saddle-Node bifurcation curves coresponding to lower limit of existance of rotational response : (a) planar excitation at different angles α with no rocking excitation (β=0); (b) vertical excitation α=0, combined with rocking excitation with different ampltudes; (c) as a function of tilt angle β for different values of α for fixed value of ω=2.

Previous studies on parametric pendulum excited on the plane [2], [3] demonstated that base excitation along an elipse is more favourable for sustaining rotation compared to excitation along tilted axis. Keeping in mind that in real application the eliptical excitation might not always be possible, here more insight is given into the tilted excitation. The general euqation of motion of the parametric pendulum excited on the plane is given by:

$$\theta'' + y'' \tan\alpha \cos\theta + (1 + y'')\sin\theta + \gamma\theta' + \eta'' = 0$$

where θ is the angular displacement of the pendulum, y is the vertical displacement of the base, α is the tilt angle between the penulum supporting guide arms and the horizontal plane, γ is the damping coefficient, η is the angle of rotation of the pendulum base. The expression for η has been derived from the geometry of the set up used in the experiments. Figure 1 shows some of the numerical results of the SN-curves for different values of the parametes α,β deciding about the direction and type of the excitation apllied on the base. All the numerical results have been verified in the experiemental study. It has demonstrated that inceasing the tilt α leads to substantial reduction in the minimum forcing amplitude required to sustain rotational motion. Secontly introducing an extra degree of freedom to the system by allowing the pednulum base to tilt, results in further reduction in the required forcing amplitude. This effect is specially visible in the low frequencies. Finally Saddle-Node bifurcation curves have been computed as a function of α and β. It has been observed that the lowest position of the curve can be achieved when β-α is maximized. This proves that the rocking excitation has a stronger effect on the stability limit of rotatioal motion than a tilt of the excitation.

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AN APPROACH TO CALIBRATION OF LOW DIMENSIONAL VIV MODELS USING CFD

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ABSTRACT

Slender marine structures such as risers, mooring cables and umbilicals are very sensitive to excitations caused by vortex shedding. These resonant vibrations know as Vortex-Induced Vibrations (VIVs) can be destructive to the structures and lead to increased fatigue damage and collapse. Semi-empirical tools are widely used by engineers to predict VIV, however, these tools are subject to many assumptions which limit their applicability. The fluid-structure interaction aspects are far from being completely understood and advanced modelling is required to investigate and predict the impact of VIV on the service life of marine structures.

We study VIVs of a slender marine structure using low dimensional models. Specifically, we perform computational fluid dynamics simulations of VIV of low-mass ratio rigid cylinder in order to calibrate existing reduced-order models based on nonlinear self-excited oscillators of Van der Pol type known as the wake oscillator models. We use the wake oscillator model that focuses on fluid-structure coupling through acceleration term [1].

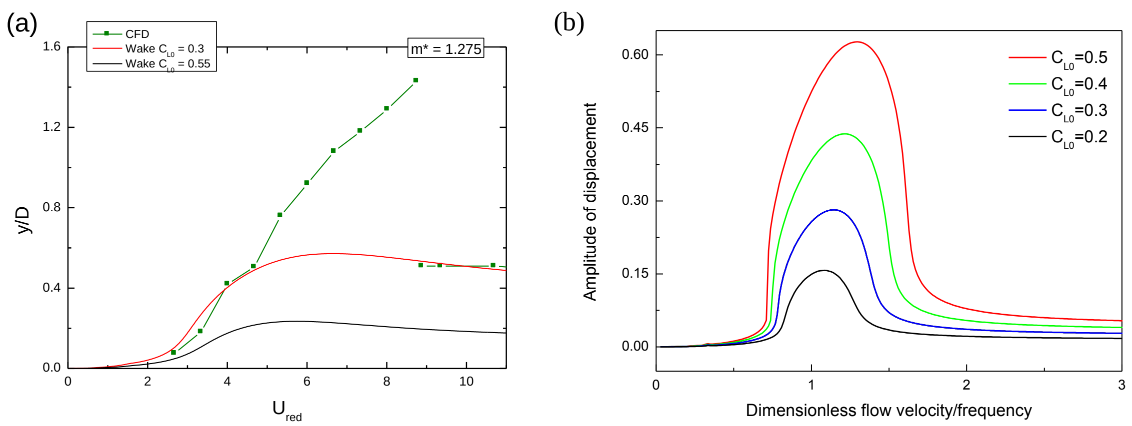

Figure 1: (a) Comparison of numerical solutions (C_{L0} = 0.3; C_{L0} = 0.55) with CFD simulations; (b) dependence of the vibration amplitude on the coefficient C_{L0} (single mode approximation).

In this work we present numerical solution of the wake oscillator model for a single mode approximation compared with results obtained with CFD (Fig. 1a) and discuss the choice of parameters for the wake oscillator model (Fig. 1b) coupled with a structure [2]. A series of CFD simulations is performed where the flow is analysed for a cylinder capable of moving in transversal and in-line directions. CFD results are verified against experimental data, where so-called "super-upper" branch of response is observed for a low-mass ratio cylinder which is free to vibrate in both directions.

The work is still in progress and the CFD model is being expanded towards multiple "strips" in order to observe higher modes of vibration of the structure.

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CHARACTERISATION OF FRICTION FORCE AND NATURE OF BIFURCATION FROM EXPERIMENTS ON A SINGLE-DEGREE-OF-FREEDOM FRICTION-INDUCED SYSTEM

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ABSTRACT

We analyze the experimental results obtained from a laboratory set-up consisting of a rigid mass connected to a fixed support through a spring and the mass is in frictional contact with a moving belt. Some of the major objectives of the experiments are to understand (i) the nature of friction-induced oscillations, (ii) the nature of bifurcation associated with frictional instability in the system, and (ii) the nature of friction force that is responsible for the oscillations observed from the experiment. The bifurcation diagram clearly demonstrates the subcriticality of the Hopf bifurcation associated with the system. The relative velocity-friction force curve shows significant hysteretic behaviour, both in the pre-sliding as well as in the pure sliding domains. This observation hints towards a dynamic or an acceleration-dependent friction model as an appropriate choice.

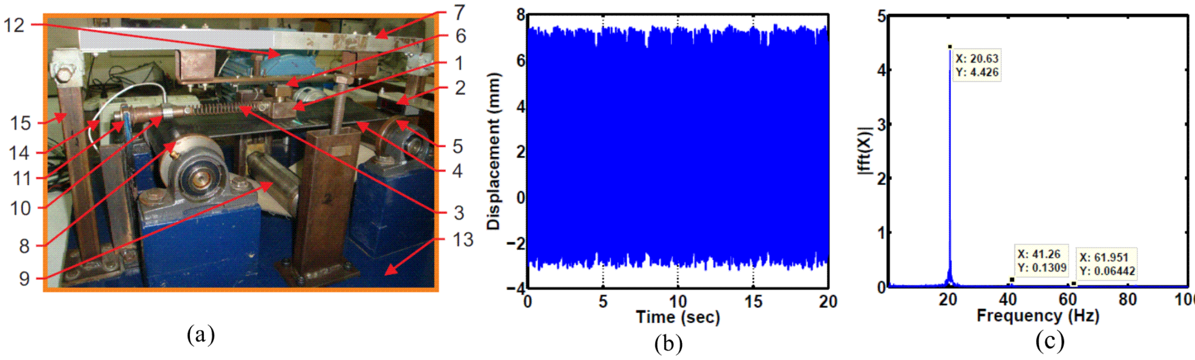

Figure 1: (a) The experimental set-up of the friction-induced system. (1) Oscillator, (2) Laser, (3) Spring, (4) Belt, (5) Driving pulley, (6) Linear bearing, (7) Top support, (8) Driven pulley, (9) Idle pulley, (10) Load cell, (11) Rigid support for holding spring and load cell, (12) AC motor, (13) Base plate, (14) VFD drive, (15) Vertical guide. (b) Time response of the oscillator for belt speed $V_b = 533$ mm/s. (c) Fast Fourier Transform of the signal shown in (b).

The laboratory set-up representing a single-degree-of-freedom friction-induced oscillation system is shown in Fig. 1a. Different components of the set-up and some of the instruments used are marked in the figure. The experimental set-up consists of a spring-mass system with the mass-(1) resting on a moving belt-(4) which is driven by an AC motor-(12). A variable frequency drive (VFD)-14 controls the speed of the motor.

The equation governing the motion of the oscillator block shown in Fig. 1a is given by

$$M\ddot{X} + C\dot{X} + KX = F(V_r), \qquad (1)$$

where M is the mass of the oscillator block, C is the damping constant, K is the stiffness of the spring, $F(V_r)$ is the friction force acting on the oscillator and V_r is the relative belt velocity. The dot in eqn. (1) represent derivative with respect to time. Several strategies have been adopted in the literature to find the friction force from eqn. (1) [1-3]. Some difficulties in measuring friction force from eqn. (1) is discussed. In our analysis, we obtain friction force from the left hand side of eqn. (1) in an alternate approach. The displacement and the acceleration of the oscillator-(1) are measured simultaneously using a single point laser sensor-(2) and an accelerometer (not shown in Fig. 1a), respectively. A low-pass Butterwoth filter is used to remove the high frequency noise from both the displacement and the acceleration signals. We then obtain the velocity of the oscillator by numerically differentiating the filtered displacement signal.

The displacement response of the oscillator for the belt speed $V_b = 533$ mm/s is shown in Fig. 1b. The Fourier transform of this displacement signal is shown in Fig. 1c showing a fundamental frequency ($f = 20.63$ Hz) and two super-harmonics. The vibration of the oscillator has modulations which might be due to a variation of the normal load resulting from the joint of the belt. However, the fast Fourier transform of the signal clearly shows a prominent fundamental frequency. Hence, the oscillation in Fig. 1b can be considered to be primarily due to some average friction force between the mass and the moving belt. Our focus in this study is to find an average friction behaviour that can approximately predict the nature of oscillations and so, we find a portion of the signal where the oscillation is almost periodic and determine the friction force from that portion only.

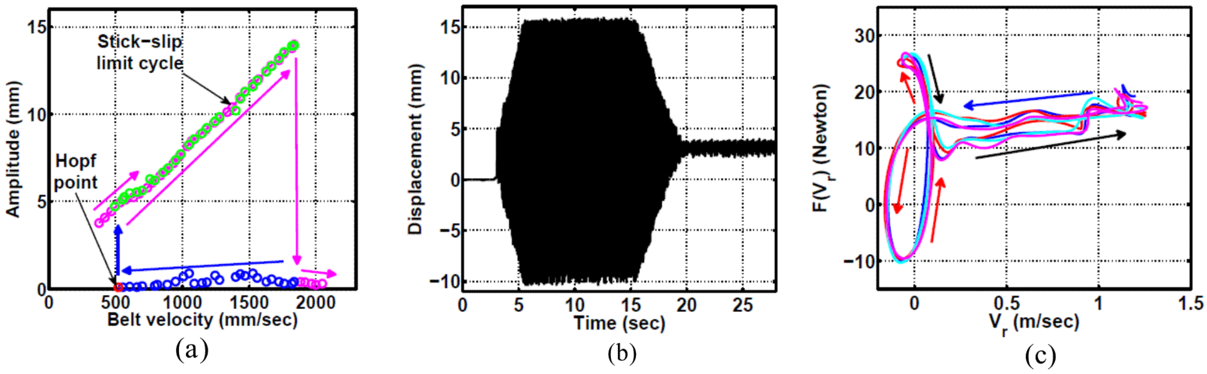

Figure 2: (a) Experimentally obtained bifurcation diagram. (b) Time response of the oscillator captured while checking the stability of the steady-state. Belt speed $V_b = 1628$ mm/s. (c) Friction force as a function of relative velocity.

To generate the bifurcation diagram, we measure the amplitudes of the oscillations obtained experimentally for different belt velocities. The amplitudes (measured from the average peak-to-peak distance) are then plotted as a function of belt velocities to obtain the bifurcation diagram which is shown in Fig. 2a. The arrows indicate the direction of changing the belt speed. Clearly, the bifurcation shown in Fig. 2a is subcritical in nature negating the possibility of a polynomial type friction model which result in supercritical Hopf bifurcation [4]. The stick-slip vibrations of the oscillator (shown by magenta circles in Fig. 2a) are captured for different belt velocities until the point at which the large amplitude vibration ceases to exist (the cyclic fold point) and small vibrations shown by the lower magenta and blue circles are obtained. These small amplitude vibrations are observed from experiments due to the variation of the normal load. The belt speed is then reduced till we get large amplitude vibrations as a result of loss of stability of the steady-state through Hopf bifurcation. The Hopf bifurcation point is indicated by a red circle in Fig. 2a. After determining the Hopf point, the experiments on finding the large amplitude vibrations are carried on again to check the repeatability of the experiments. The green circles in Fig. 2a are the amplitudes of the friction-induced vibrations captured during these experiments.

After capturing all the large amplitude vibrations, the stability of the steady-states are then checked again for belt velocities lower than the belt velocity corresponding to the cyclic fold point which are indicated by the blue circles. Special care is taken while examining the stability of the steady-states for this case. During this experiment, the oscillator is first allowed to vibrate at a larger amplitude for a particular belt velocity before forcing it to come to the steady-state and the subsequent evolution of the vibrations determine the stability of the steady state. One such displacement response for belt velocity $V_b = 1628$ mm/s during this experiment is shown in Figs. 2b which indicates that the steady-state is stable in this case.

The friction force measured using the strategy described earlier is plotted against the relative belt velocity in Fig. 2c which shows significant hysteretic behaviour, both in the pre-sliding as well as in the pure sliding domains. Four different friction curves shown in different colors in Fig. 2c correspond to four different cycles of the periodic portion of time response chosen for determining the friction force. The downward arrow in black indicates the decrease of the friction force from the static to kinetic friction. The two black arrows correspond to the acceleration phase of the oscillator, whereas the blue arrow is for the deceleration phase. The red arrows show the direction in the pre-sliding regime. The nature of the friction curves predicts that a dynamic or an acceleration-dependent friction model is suitable in depicting the nature of oscillations observed experimentally.

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ATTRACTOR RECONSTRUCTION FOR PARAMETER IDENTIFICATION IN AN IMPACT OSCILLATOR

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ABSTRACT

Prior knowledge of the rock mechanical properties such as the effective stiffness could highly improve drilling performance, resulting in increased penetration rate. This is especially true when a technology such as resonance enhanced drilling is used, in which the operating parameters are adjusted to fit the requirements for each formation. Thus far, there are two techniques used in drilling industry to identify the type the formations in the wall lithology, namely; seismic surveys and wall logging. Neither is accurate enough to sense small changes in the formation properties. Hence, there is lack of methods that can provide information about the properties of formation such as its effective stiffness in real time. This was the motivation behind studying the dynamical behaviour of a simplified drilling system with the aim of obtaining information about mechanical properties of the formations from the dynamical behaviour of the drill-bit response. For this purpose a simple impact oscillator which has some non-smoothness is used to investigate the effect of different stiffness ratios [1]. The system is designed to represent the interaction between drill-bit and the rock formation in a simplified way in which the effect of the extra degrees of freedom is excluded. A systematic non-linear time series analysis is used to study the impact behaviour and its relationship with formation mechanical properties.

In the proposed method acceleration time histories are used because impacts are well pronounced compared to other measurements. Phase-spaces are reconstructed using Takens embeddings [2] for several time histories. Stationarity and determinism is demonstrated for the data. From the reconstructed phase spaces, it is clear that there is a relationship between the effective formation stiffness and the geometry of the attractor. Figure 1 provides an example of this relationship; for high stiffness ratios a sharper inclination of the part of the attractor which deviates from the plane is clearly seen. Statistical analysis of the tangent vectors reveals this correlation. In one proposed method, the vector products of successive tangent vectors are used to estimate deviation from the linear plane, although results from this approach prove to be sensitive to noise. A second approach is proposed, which uses an averaged tangent vector for data before and after the impact in a 2-D projection of the phase-space. A least squares technique is used estimate the best to fit through the data. This approach is less sensitive to noise in comparison with the previous method.

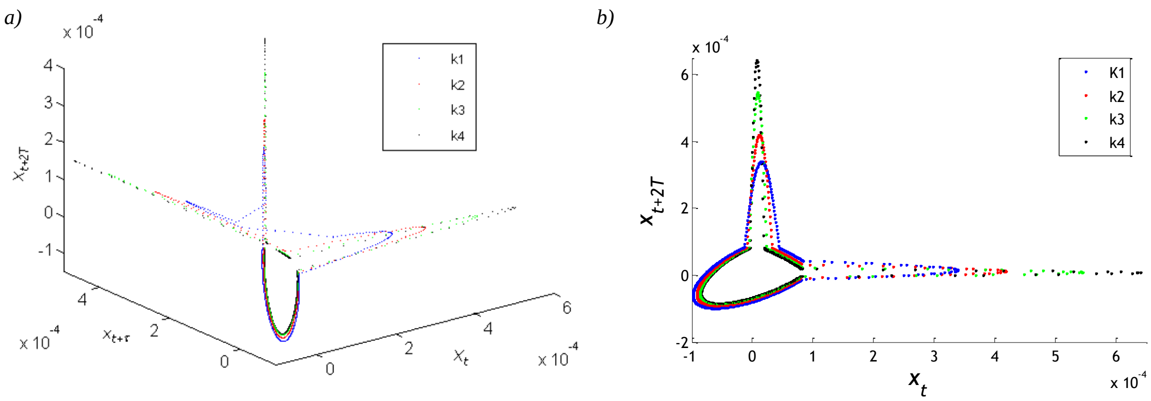

Figure1. *a)*: 3D plot of the reconstructed phase-spaces for four acceleration time histories with different stiffness ratios. *b)* X-Z projection of the phase-space in which the relationship can be observed. k*n* is the stiffness ratio (k1< k2< k3< k4).

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EXPERIMENTAL STUDY OF DRILL-STRING VIBRATIONS

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ABSTRACT

Dynamic effects such as uncontrolled vibrations are harmful to the drilling process as they may cause premature wear and damage of the drilling equipment, which eventually results in expensive failures. These include bit-bounce, stick-slip, forward and backward whirl, together with direct and parametric coupling between axial, torsional and lateral vibrations. Drill-collars and drill-pipes are subjected to the most damaging vibrations. The Bottom Hole Assembly (BHA) not only influences significantly the dynamic responses of the entire drill-string, but it is also the location of most failures [1].

Vibrations often induce well-bore instabilities that can worsen the condition of the well and reduce the directional control. In the last decade, the dynamics of drilling system has been studied numerically and experimentally however, the mechanism of most experimental rigs which are reported in the literature, are not based on real drilling, for example, Mihajlovic *et al*. [2] and Khulief *et al.* [3], use brake systems instead.

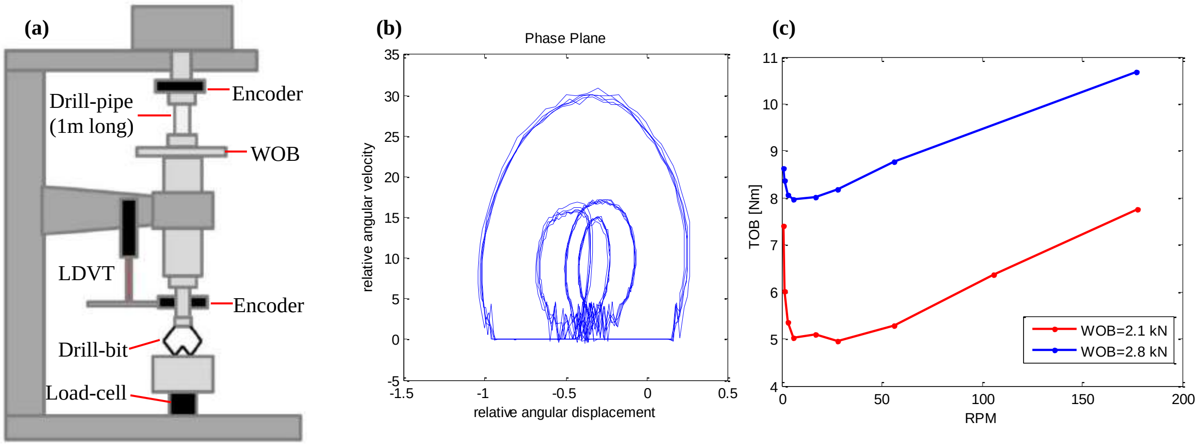

Figure 1: (a) Schematic of the drilling rig (b), phase plot showing stick-slip when in sandstone (c) TOB versus RPM during drilling sandstone for two different WOB, 2.1 and 2.8 kN.

In this regard, an experimental drilling rig has been designed and constructed in order to provide a fuller understanding and ultimately suppression of stick-slip oscillations, drill-bit bouncing and whirling. As it is shown in Fig. 1(a), a motor is connected to the drill pipe and the rotary force is transmitted to the bit through a drill-pipe, BHA and bit holder. The angular velocity of the motor is adjustable and it is measured by an encoder on the top of the drill-pipe. Also angular velocity of drill-bit is measured by another encoder connected to the BHA. Horizontal and vertical forces and torque coming from the drill-bit to the rock are detected by a load-cell placed under the rock. The rate of penetration of the bit into the rock is measured by a linear variable differential transducer (LVDT) attached to the BHA. Velocity of drilling, type of drill-pipe and drill-bit, Weight On Bit (WOB) and rock are changeable and consequently, in addition of main purpose, this will allow the identification of minor failures, dynamic characteristics of drill-pipes and BHA, rock models and drilling conditions.

We observe the stick-slip oscillations with the realistic drilling experiment setup. Fig. 1(c) presents a phase plot of an interesting onset of stick-slip oscillations from much smaller oscillations. In order to model these phenomena, the mechanical characteristics of the drill-bits have been determined. Sample of the experimental results for a 3-7/8″ PDC drill-bit are presented in Fig. 1(c), where two curves were taken for 2.1 and 2.8 kN WOB respectively. Therefore based on the mechanical characteristics of the drill-bit, we model the stick-slip phenomena and compare it with experimental results.

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EXPERIMENTAL STUDY OF CONTROL METHODS FOR MAINTAINING ROTATION OF PARAMETRIC PENDULUM

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ABSTRACT

Previous theoretical and experimental investigations [1]–[4] have analyzed the dynamics of parametric pendulum regarding its application for wave energy extraction. Besides, a few attempts have been recently made to maintain the rotational motion of the parametric pendulum based on Time-Delayed Feedback Control method (TDF) [5-6]. This paper focuses on the experimental application of different start-up control methods, as well as on investigation of the performance of TDF under varying excitation parameters and noise. The additional torque (***u***) coming from the servomotor can be included in non-dimensional equations of the parametric pendulum on a flexible support, based on the model presented in [3]:

$$y'' + \gamma(y' + p\omega \sin(\omega\tau)) + \alpha(y - p\cos(\omega\tau)) + a(\theta'' \sin\theta + \theta'^2 \cos\theta) = 0$$
$$\theta'' + (1 + y'')\sin\theta + \gamma_\theta \theta' + u = 0$$

where γ is the damping coefficient in the vertical direction, γ_θ is the damping on the shaft, α is the stiffness coefficient of the support, p is the forcing amplitude, ω is the forcing frequency, τ is the non-dimensional time and a is a mass ratio. The modified control signal is given by:

$$u = k\,\mathrm{sgn}(\theta')(\theta(t-\Gamma) - \theta(t) + 2\pi)$$

where Γ is the period of the desired UPO (unstable periodic orbit), θ is the angular displacement of the pendulum, and k is the gain of the controller.

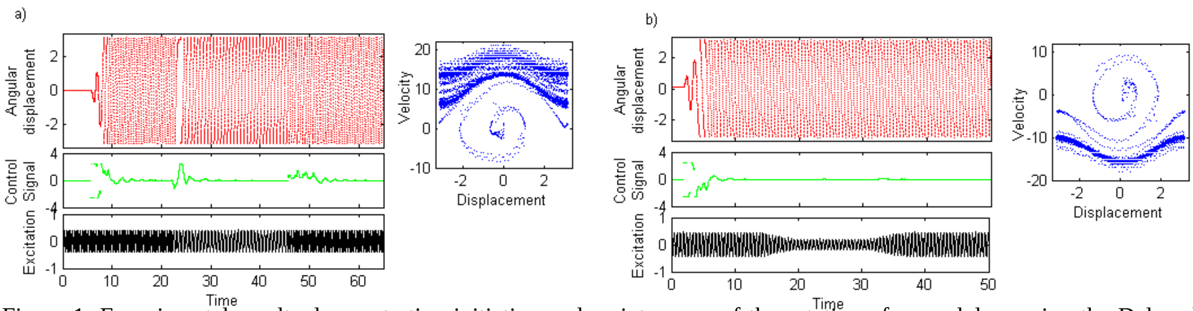

Figure 1: Experimental results demonstrating initiation and maintenance of the rotation of a pendulum using the Delayed-Feedback with multi-switching method. In (a) frequency of excitation is changed continuously from 2.5 Hz to 1.4 Hz and back to 2.5 Hz, while the controller maintains the rotations. In (b) amplitude of excitation varies continuously from 0.450 V to 0.150 V and back to 0.450 V with the same result.

The time history, phase plane and control signal of the two experiments are shown in Fig. 1. In Fig. 1(a) the frequency of excitation was changed continuously, in Fig. 1 (b) the amplitude of excitation has been varied while the rotations were maintained. In this paper we will demonstrate experimentally the effectiveness of the delayed feedback control in maintaining rotational response while different perturbations of the excitation signal occur, including time varying forcing frequency and amplitude as well as additional noise present in the excitation or control signal.

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DYNAMICAL MODEL AND ANALYSIS OF NON-HARMONIC VIBRATION CONVEYOR

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ABSTRACT

The non-harmonic vibration conveyor is widely used in Electrical-Arc-Furnace system to convey steel scrape[1], for which the dynamical analysis is important to application[2-3].

The draft of two-harmonic vibration conveyor is shown in Fig. 1. The dynamical model for the single particle is shown in Fig. 2 (a), and the horizontal excitation force is described in Fig.2 (b). If the reaction force of bulk materials is ignored, the motion of vibrating body can be expressed as

$$\begin{aligned}\ddot{x}_0(t) &= A\omega^2\cos(\omega t+\theta)+A\alpha\beta^2\omega^2\cos(\beta\omega t+\beta\theta+\varphi) \\ \dot{x}_0(t) &= A\omega\sin(\omega t+\theta)+A\alpha\beta\omega\sin(\beta\omega t+\beta\theta+\varphi) \\ x_0(t) &= -A\cos(\omega t+\theta)-A\alpha\cos(\beta\omega t+\beta\theta+\varphi)\end{aligned} \quad (1)$$

where A, ω, θ refer to the amplitude, frequency and initial phase of low frequency main vibration and α, β, φ refer to the amplitude ratio, frequency ratio and phase difference between the main vibration with low frequency and the minor vibration with high frequency.

By introducing the relative motion $x = x_1 - x_0, \dot{x} = \dot{x}_1 - \dot{x}_0$ and symbolic function[4], the dry friction force between the vibrating body and single particle can be comprehensively defined as

$$F_r = \begin{cases} -\operatorname{sgn}(\dot{x})\mu_d mg & \dot{x} \neq 0 \\ -\operatorname{sgn}(\ddot{x}_0)\mu_s mg & \dot{x}=0, |\ddot{x}_0| > \mu_s g \\ m\ddot{x}_0 & \dot{x}=0, |\ddot{x}_0| \leq \mu_s g \end{cases} \quad (2)$$

where g is the acceleration of gravity; μ_d, μ_s are respectively the dynamic friction coefficient and the static friction coefficient between the vibrating body and single particle.

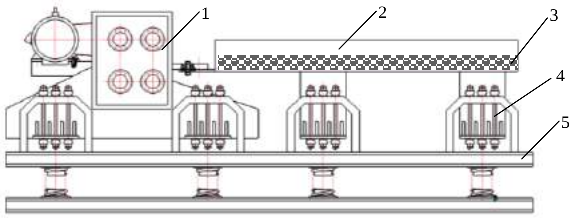

Fig. 1 Draft of non-harmonic vibration conveyor
1 exciter; 2 chute; 3 bulk materials; 4 hanging rod; 5 base.

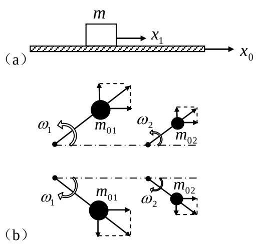

Fig. 2 (a) Dynamical model for single particle
(b) Excitation force for vibrating body

The conveying velocity is a key indicator to the conveying effect. If only the vibration parameters are considered to the conveying effect, the dynamical model can be built as a single-degree-of-freedom system when the bulk materials are simplified as single particle. If the other parameters are considered to the conveying effect, including the property of material and the contact property between particle and particle, the bulk materials must be built as multi-particle dynamical model with DEM (Discrete Element Method). In this paper, the numerical simulation results are compared for single particle and multi-particle dynamical model, and the effect on conveying velocity are studied for some parameters.

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BIFURCATION AND QUENCH CONTROL OF GRINDING CHATTER

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ABSTRACT

The nonlinear behaviors of grinding chatter were investigated in [1], with its dynamic mode shown in Figure 1(a). As seen in Figure 1(b), the grinding chatter is induced by the regeneration of the wheel and the workpiece surfaces. To clarify the mechanism of regenerative chatter, bifurcation analysis is carried out to track the periodic solutions, with the corresponding bifurcation diagram constructed in Figure 2(a). Clearly, the pattern presents a subcritical Hopf bifurcation, which introduces large-amplitude chatter and shrinks the unconditional stable region. To transform this unwanted bifurcation, cubic nonlinear control is employed, where the relative velocity between the workpiece and the wheel is used as a feedback. With the increase of the feedback gain, the bifurcation is transformed not only locally (see Figure 2(b)) but also globally (see Figure 2(c)). Next, to further suppress the chatter amplitude, quench control is adopted as well [2], where a periodic external vibration is applied on the grinding wheel. Basically, the idea of quench control is replacing the grinding chatter by another small-amplitude forced vibration. With the increase of the excitation amplitude, the grinding chatter is finally suppressed. Its effect is illustrated in Figure 3. As seen in Figure 3(b), bifurcation control only decreases the chatter amplitude, while the quench control changes the vibration frequency as well. This phenomenon illustrates that the original grinding chatter is successfully suppressed by the quench control.

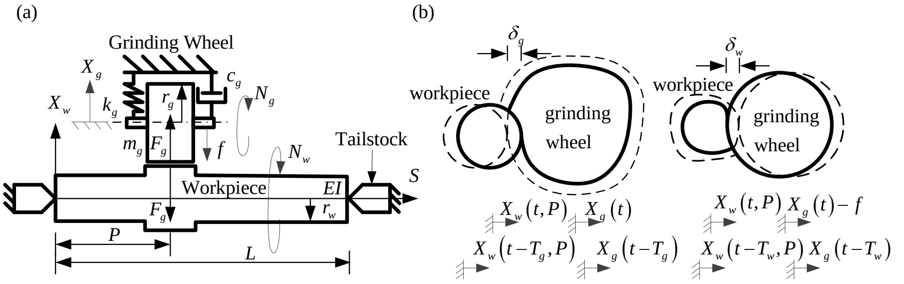

Figure 1: Schematics of the (a) plunge grinding process and the (b) regenerations in the wheel and workpiece surfaces.

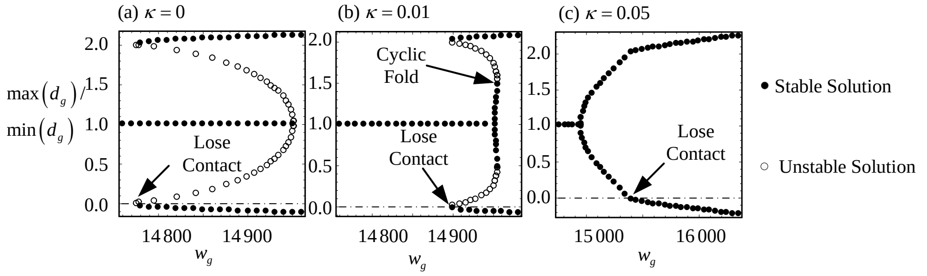

Figure 2: Bifurcation diagrams of grinding depth (minimum and maximum values of d_g during the grinding): (a) no control (b) bifurcation control with gain $\kappa = 0.01$ (c) control with gain $\kappa = 0.05$.

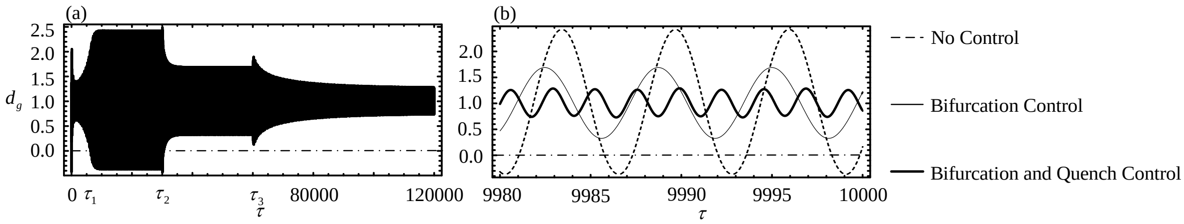

Figure 3: Time series: (a) bifurcation and quench control are applied at τ_2 and τ_3 respectively; (b) different steady states.

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DYNAMICS OF COMPOUND BURSTING COMPOSED OF SEVERAL BURSTS AND SUBTHRESHOLD OSCILLATION

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ABSTRACT

Episodic or compound bursting is a very complex bursting pattern with a transition between a burst episode composed of several bursts and a relatively long rest state. The minimal and generic phantom bursting model [1] is used to produce compound bursting as presented in Fig.1, when two slow variables with different time scales s1 and s2 commonly or separately play different roles in different parts. It is obviously not enough to explain dynamic behavior of the complex compound bursting only considering the slower slow variable s2[2]. Therefore, the different fast/slow analysis for the moderate and the slower slow variables s1 and s2 in Fig.2 and Fig.3, respectively, are given in a three-dimensional space to reveal complex dynamic of the compound bursting. At first, the beginning and the ending of every burst in burst episode are decided by the fold bifurcation and the saddle homoclinic bifurcation for the moderate slower variable s1, where the little tilted form at the start is resulted from locating in the C-shaped stable limit cycles. Nonetheless, the burst episode is terminated by common effects of the different saddle homoclinic bifurcations for s1 and s2. Then, the first burst exactly reaches the stable limit cycle for the bifurcation parameter s2, when the stable limit cycle for the bifurcation parameter s1 hits saddle on the middle branch. So the first burst existing in stable limit cycle for s2 have many spikes but the others locating in the unstable limit cycles for s2 are not able to continue and become the short bursts, which can explain different occurrence of the long burst and the short bursts. Finally, during the long silent phase the property of the stable focus on the lower branch for the bifurcation parameter s2 accounts for the subthreshold oscillation with large amplitude on both sides and small amplitude on the middle.

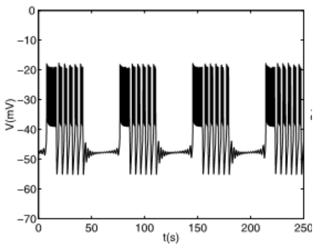

Fig.1

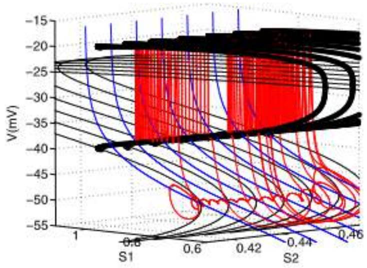

Fig.2

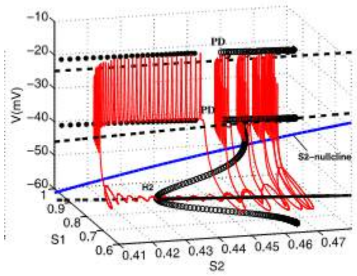

Fig.3

Table.1

g_c	s_2 for TR	s_2 for HC	The difference between them
1.6	0.4596	0.4745	0.0149
1.95	0.4654	0.4787	0.0133
2.5	0.4742	0.4827	0.0085
3.3	0.4853	0.4916	0.0063

Furthermore, synchronization of bursting activity plays an important role in the insulin secretion. Using the minimal phantom bursting model of two electrical coupling pancreatic beta-cells, we examine for the first time compound bursting synchronization phenomena with anti-phase spikes. Since the total time duration of all the short bursts is decided by the existing range of the unstable limit cycle between the TR bifurcation point and the saddle homoclinic point. So the decrease of the difference between these two points for the increasing coupled strength as shown in Table.1 leads to the decreasing durations of all these short bursts. All above can explain the number of the short bursts decreasing from 4 to 3, 2 and 1, in consideration of rough invariability of the time duration of every short burst and the time intervals between the short bursts.

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A NONLINEAR MODEL OF BALANCING HUMAN STANDING

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ABSTRACT

Previous studies of human self-balancing have validated the modeling of human body as an inverted pendulum, and considered time delays in the neural pathways to be key factors which affect the balancing greatly [1]–[4]. A pivot-fixed inverted pendulum with delay feedback control is used to model balancing without considering that the pivot is moving [4]. This paper proposes a new nonlinear model for the mechanical structure of balancing, considering both delay effect and pivot-shift. In stable posture of balancing, the center of gravity (COG) of human body is within the base of support (BOS), thus there is a stable domain of inclination (SDOI) of human body. The new model introduces time delay in the pressures and torques generated by foot, use two critical postures to replace all the postures with varying pivots on the foot, to calculate the SDOI. By applying some assumptions and reductions, the dynamic nonlinear model can be described as follow:

$$\begin{cases} \ddot{\beta}+(g/l_{OA})\cos(\beta+\alpha)=k_1\beta(t-\tau)+k_2\dot{\beta}(t-\tau), & \beta>0 \\ \ddot{\beta}-(g/l_{OB})\cos(-\beta+\alpha)=k_1\beta(t-\tau)+k_2\dot{\beta}(t-\tau), & \beta\le 0 \end{cases} \tag{1}$$

where β is the inclination of the body, g is the acceleration of gravity, l_{OA}, l_{OB} are the distances between the two critical pivots and the COG, α is a constant related to the distance between two critical pivots of human body, constants k_1, k_2 are the gains of the delayed PD control, τ is the time delay.

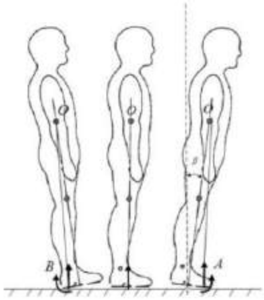

Figure 1: Postures of human self-balancing, the left and right postures represent the critical postures with critical pivots.

Note that Equation (1) is not smooth and $\beta=0$ is not an equilibrium point while actually it should be, so we use a continuous function $\tanh(n\beta)$ with a large enough constant n to approximate the sign function. In addition, let $l_{OA}=l_{OB}=l_0$ for simplicity, then we have:

$$\ddot{\beta}-(g/l_0)\sin(\beta)\sin(\alpha)+(g/l_0)\tanh(n\beta)\cos(\beta)\cos(\alpha)=k_1\beta(t-\tau)+k_2\dot{\beta}(t-\tau) \tag{2}$$

with $\beta\in(-\pi/2,\pi/2)$. Equation (2) has one more equilibrium point as $\beta=0$ compared to equation (1). This model has some basic properties during balancing.

We analyze how the SDIO varies with α and prove that if the distance between the two critical pivots becomes smaller, the SDIO shrinks too. We do some numerical calculation for the delay effect and find that the SDIO shrinks as the delay value grows larger, sometimes even smaller than cases without control. These results can be used to explain why the elders whose time delays are usually larger have greater possibilities to fall over than the younger people, and they can also be used to explain balancing sticks, of which the contact surfaces such as cylinders are large and should not be ignored, using our palms or fingertips. And the numerical results show also that oscillations appear when introducing time delay, which does not exist in delay-free balancing and may explain why people sway slowly when they are in quiet standing.

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DESIGN OF THE DELAYED OPTIMAL FEEDBACK CONTROL FOR LINEAR SYSTEMS WITH MULTIPLE DELAYED INPUTS

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ABSTRACT

The optimal control of linear systems has been a major control strategy in engineering applications over the past few decades. Dynamic Programming Method and Pontryagin's Maximum Principle are two most popular methods for solving optimal control problems. When time delays are involved in the control systems, the control design becomes challenging. For some simple cases, the controlled system with a single input delay is converted into a delay-free system by using an integral transformation [1], so that the classical optimal control theory can be used for the design problem. Such a transformation, however, does not work for the controlled systems with multiple delayed inputs. The methods based on Pontryagin's Maximum Principle work but they may yield just a local optimum, not the overall optimum. In our previous paper [2], we observed that under quadratic performance criterion, the input delay does not affect the optimal control directly for linear time-invariant systems with a single input delay. Based on this observation, we proposed a simple design formula of the delayed optimal control in closed form, so that the control design can be carried out straightforwardly step-by-step. In this paper, we further investigate the design problem for linear time-varying systems with multiple delayed inputs. By using the Dynamic Programming Method, we prove that the delayed optimal control can be determined similarly as done in [2].

Consider a linear time-variant system with multiple delayed inputs

$$\dot{x}(t) = A(t)x(t) + \sum_{i=1}^{N} B_i(t)u_i(t-\tau_i) \tag{1}$$

with the initial condition $x(0) = x_0$. Here, $x(t) \in \mathbb{R}^n$ is the system state and is completely measurable, and $u_i(t-\tau_i)$ is the delayed control variable which takes place when $t \geqslant \tau_i$. Without loss of generality, we assume that the delays are in an increasing order: $0 \le \tau_1 < \tau_2 < \cdots < \tau_N$. The quadratic performance criterion to be minimized is defined as follows

$$J = \frac{1}{2}x^{\mathrm{T}}(t_f)Mx(t_f) + \frac{1}{2}\int_0^{t_f}\left[x^{\mathrm{T}}(t)Q(t)x(t) + \sum_{i=1}^{N} u_i^{\mathrm{T}}(t-\tau_i)R_i(t)u_i(t-\tau_i)\right]\mathrm{d}t \tag{2}$$

where M, $Q(t)$, $R_i(t)$ are the cost weighting matrixes, and t_f is the terminal time. By using the classic LQ optimal control theory and the Dynamic Programming Method, the delayed optimal feedback controls of the linear system (1) that minimize the quadratic performance criterion (2) are found to satisfy

$$u_k^*(t-\tau_k) = -\Phi_i^{-1}B_k(t)^{\mathrm{T}}P_i(t)x(t), t \in [\tau_i, \tau_{i+1}), k = 1, 2, \cdots, i, i = 1, 2, \cdots, N \tag{3}$$

where $\tau_{N+1} = t_f, P_{i+1}(t_f) = M$, and $P_i(t) \in R^{n\times n}$ is the positive definite solution of the Riccati differential equations described by

$$\dot{P}_i(t) = -P_i(t)A(t) - A(t)^{\mathrm{T}}P_i(t) + P_i(t)\bar{B}_i(t)\Phi_i^{-1}\bar{B}_i(t)^{\mathrm{T}}P_i(t) - Q(t), i = 1, 2, \cdots, N \tag{4}$$

subject to $P_i(\tau_{i+1}) = P_{i+1}(\tau_{i+1})$, and $\bar{B}_i(t) = [B_1(t), B_2(t), \cdots, B_i(t)], \Phi_i = \mathrm{diag}(R_1(t), R_2(t), \cdots, R_i(t))$. Due to (3), the quadratic performance criterion (2) can be simplified to the following form

$$J = \tfrac{1}{2}\int_0^{\tau_1} x^{\mathrm{T}}(t)Q(t)x(t)\mathrm{d}t + \tfrac{1}{2}x(\tau_1)^{\mathrm{T}}P_1(\tau_1)x(\tau_1).$$

Moreover, the optimal state expression in the whole interval can be obtained, by calculating the optimal state in $[0, \tau_1]$, $[\tau_1, \tau_2]$, $\cdots$, respectively. And the delayed optimal feedback gains K_{di} can be obtained from

$$u_k^*(t-\tau_k) = -K_{dk}x(t-\tau_k) = -\Phi_i^{-1}B_k^{\mathrm{T}}P_i(t)x(t), t \in [\tau_i, \tau_{i+1}), k = 1, 2, \cdots, i, i = 1, 2, \cdots, N. \tag{5}$$

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PERIODIC BIFURCATION ANALYSIS OF FRACTIONAL DERIVATIVE AND DELAY SYSTEMS

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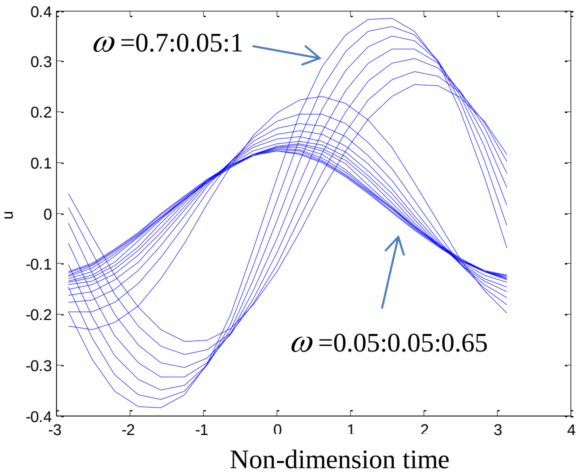
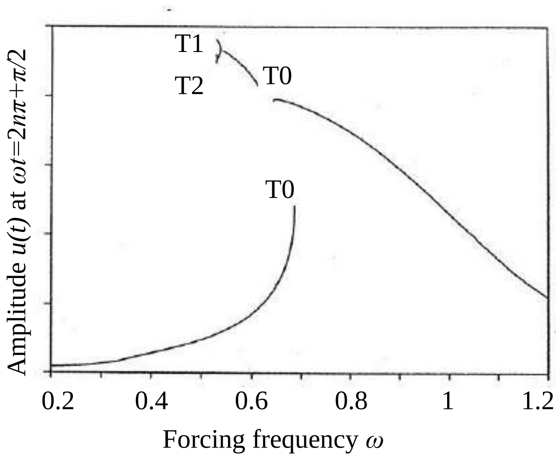
ABSTRACT

The harmonic balance (HB) method substitutes the time variable by Fourier coefficients and therefore replaces the nonlinear ODE to nonlinear algebraic equations. The bifurcation analysis can then be carried out on the nonlinear algebraic equations. However, HB can hardly deal with irrational nonlinearity, fractional derivatives and time delay and the number of effective Fourier terms cannot be determined in the beginning. Changing the number of terms invokes complete recalculation. A new spectral matrix for the steady state analysis of periodic system is proposed. New techniques include (1) replacing any time dependent variable by a number of points along the periodic orbit; (2) representing any integer or fractional derivatives by simple matrices; (3) choosing the points optimally for spectral convergence; (4) replacing time delay by matrices; (5) representing nonlinear damping and mass by Kronecker products; (6) analyzing the resulting algebraic equations by LS reduction. It is found that the method is very effective for investigating systems having time delay, fractional derivatives and nonlinear inertia and damping.

Example
Consider undamped Duffing equation: $F = u'' + 2\mu u' + ku + \alpha u^3 - f\cos\omega t = 0$. Let $\tau = \omega t$ then $F = \omega^2\ddot{u} + 2\mu\omega\dot{u} + ku + \alpha u^3 - f\cos\tau = 0$. Let $\mu = 0.2; k = 1; \alpha = -4; f = 0.115$. After discretization in time $\mathbf{F} = \omega^2\mathbf{D}^2\mathbf{u} + 2\mu\omega\mathbf{D}\mathbf{u} + k\mathbf{u} + \alpha\mathbf{u}.\wedge 3 - f\mathbf{c} = 0$, where $\mathbf{D}$ is the periodic differentiation matrix and $\mathbf{u} = col\{u(t_1),\cdots u(t_n)\} = col\{u_1,\cdots u_n\}$ (unknown) and $\mathbf{c} = col\{c_1,\cdots c_n\}$ (given). The gradient is given by $\mathbf{F_u} = \omega^2\mathbf{D}^2 + 2\mu\omega\mathbf{D} + k\mathbf{I} + 3\alpha\,\mathrm{diag}(\mathbf{u}.\wedge 2)$ so that Newton iteration and bifurcation can be performed. The solutions using N=20 using ω =0.05:0.05:1 are given below where bifurcation between ω =0.65 and 0.7 is obvious.





An important novel contribution is that we can prove that $\frac{d^\beta u}{dt^\beta}$ corresponds to $\mathbf{D}^\beta\mathbf{u}$ where β is the fractional order so that fractional derivatives can be treated easily. We shall cover time-delay and other nonlinear effects.

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EXACT SOLUTIONS FOR DISCRETE BREATHERS IN FORCED – DAMPED CHAIN

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ABSTRACT

Exact solutions for symmetric discrete breathers (DBs) are obtained in forced – damped linear chain with on-site vibro-impact constraints [1-3]. The damping in the system is caused by inelastic impacts; the forcing functions should satisfy conditions of periodicity and antisymmetry. Global conditions for existence and stability of the DBs are established by combination of analytic and numeric methods. An example of such "existence – stability" diagram is presented in Fig. 1.

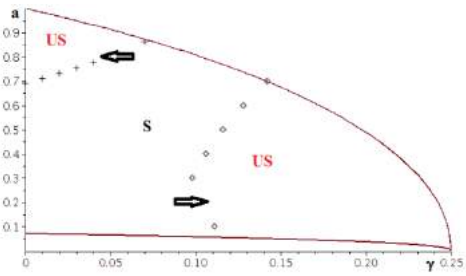

Figure 1. Zones of existence and stability for the symmetric DB on the parameter plane for two different values of the restitution coefficient. Crosses denote the pitchfork bifurcation line and diamonds denote the line of Neimark – Sacker bifurcation.

The DB can lose its stability either by pitchfork, or through Neimark – Sacker bifurcations. The pitchfork bifurcation is related to internal dynamics of each individual oscillator. It is revealed that the coupling can suppress this type of instability. To the contrary, the Neimark – Sacker bifurcation occurs for relatively large values of the coupling, presumably due to closeness of the excitation frequency to a boundary of propagation zone of the chain. Both bifurcation mechanisms seem to be generic for considered type of forced – damped lattices. Some unusual phenomena, like non-monotonous dependence of the stability boundary on the forcing amplitude, are revealed analytically for the initial system and illustrated numerically for small periodic lattices. The latter phenomenon is illustrated in Fig. 2 a-c.

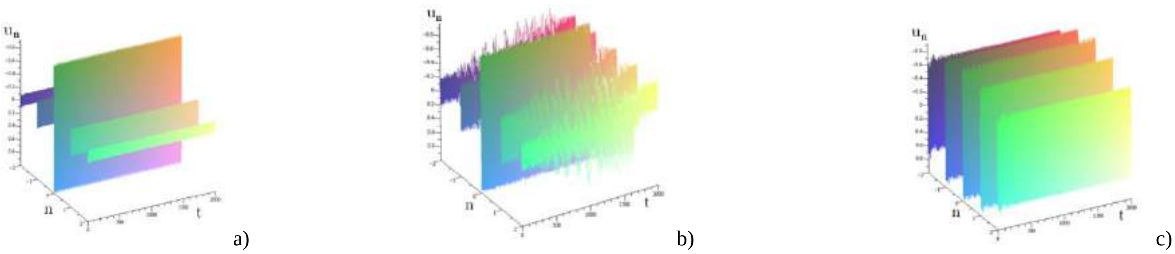

Figure 2. Numeric simulation of the DBs in small system with smoothened potentials and damping; the figures correspond to growing external forcing. Time series for 5 central particles ($-2 \le n \le 2$) are presented.

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STABILITY ANALYSIS FOR INTERMITTENT CONTROL OF CO-EXISTING ATTRACTORS

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ABSTRACT

In the recent times, control of nonlinear dynamical systems has been among the most active fields of research due to its diverse applications in engineering. Most of the existing work has been focused on controlling chaos including stabilizing a desired unstable periodic orbit embedded in a chaotic attractor, exploiting chaos' characteristics for its control, and synchronizing two identical/different chaotic systems. However, little attention has been paid to control of systems that exhibit multistability for a given set of parameters, despite the fact that they are observed abundantly in different fields of science and engineering.

This paper continues the work of intermittent control in [1] for a class of non-autonomous dynamical systems that naturally exhibit co-existing attractors, often termed multistability. The central idea shown in Fig. 1 is based on understanding of the system's basins of attraction where the control action is switched on only in the vicinity of the crossings of the actual and the desired trajectories shown in Fig. 1 (e). This paper focuses on the stability analysis of a single control action, and explores its minimum-energy control from an actual attractor to the fractal basin of a desired attractor by calculating the eigenvalues of its Jacobian matrix.

The method we proposed is applied to an impact oscillator which exhibits multistability at a specified set of parameters. Numerical results will be presented for all four cases of a single control action: (1) an unconstrained control action; (2) a single constrained control action to the main desired basin; (3) a single constrained control action to the closest fractal desired basin; (4) a single constrained control action failed to switch the system state. For each case, corresponding condition is found for the amplitude of the single control action, and finally, a minimum-energy control action is obtained.

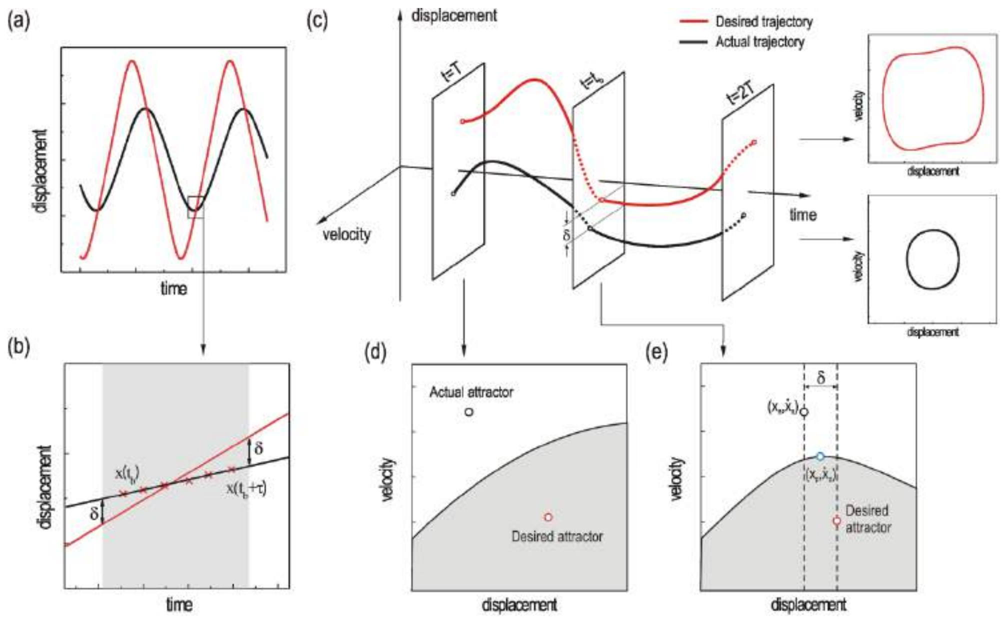

Figure 1. (a) Trajectories of the displacements of actual (black line) and desired (red line) trajectories in the time domain; (b) A neighbourhood of the crossings of actual and desired trajectories which satisfy $|e_1|\leq\delta$ (t_b is the time when the neighbourhood condition is initially satisfied, and τ is the duration of the control action); (c) Schematics of actual and desired trajectories showing the cross section in time where the neighbourhood condition is satisfied, and additional windows showing both trajectories on the phase plane; (d) Basins of attraction of the two co-existing orbits; and (e) Basins of attraction re-calculated at the phase shift where the neighbourhood condition is satisfied (adopted from [1])

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CRISIS IN CHAOTIC PENDULUM WITH FUZZY UNCERTAINTY

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ABSTRACT

Physical systems are often subjected to noisy excitations and parametric uncertainties. The interplay between noise uncertainty and nonlinearity of dynamical systems can give rise to unexpected global changes in the dynamics. In general, noise is theoretically modeled as a random variable and a fuzzy set leading to the two categories of fuzzy and stochastic dynamics. An important problem is to understand the underlying mechanism for various bifurcations and complicated phenomena in noisy (fuzzy and stochastic) dynamics. Fuzzy crises occur in chaotic pendulum in the presence of fuzzy uncertainty

$$\frac{d^2x}{dt^2} + k\frac{dx}{dt} + \sin x = S\sin\omega t \quad (1)$$

where S is a fuzzy parameter of the forcing amplitude with a triangular membership function,

$$\mu_S(s) = \begin{cases} [s-(s_0-\varepsilon)]/\varepsilon, & s_0-\varepsilon \le s < s_0 \\ -[s-(s_0+\varepsilon)]/\varepsilon, & s_0 \le s < s_0+\varepsilon \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

$\varepsilon > 0$ is a parameter characterizing the intensity of fuzziness of S and is called fuzzy noise intensity. s_0 is the nominal value of S with membership grade $\mu_S(s_0) = 1$. Here x represents the angle from the vertical of a pendulum subject to an external torque which varies sinusoidally in time with frequency ω and the fuzzy forcing amplitude S.

By means of the fuzzy generalized cell mapping method[1-2]. A fuzzy chaotic attractor is characterized by its topology and membership distribution function. A fuzzy crisis implies a simultaneous sudden change both in the topology of a fuzzy chaotic attractor and in its membership distribution. It happens when a fuzzy chaotic attractor collides with a regular or a chaotic saddle. Two types of fuzzy crises are specified, namely, boundary and interior crises shown in Figure 1. In the case of a fuzzy boundary crisis, a fuzzy chaotic attractor disappears after a collision with a regular saddle on the basin boundary. In the case of a fuzzy interior crisis, a fuzzy chaotic attractor suddenly changes in its size after a collision with a chaotic saddle in the basin interior.

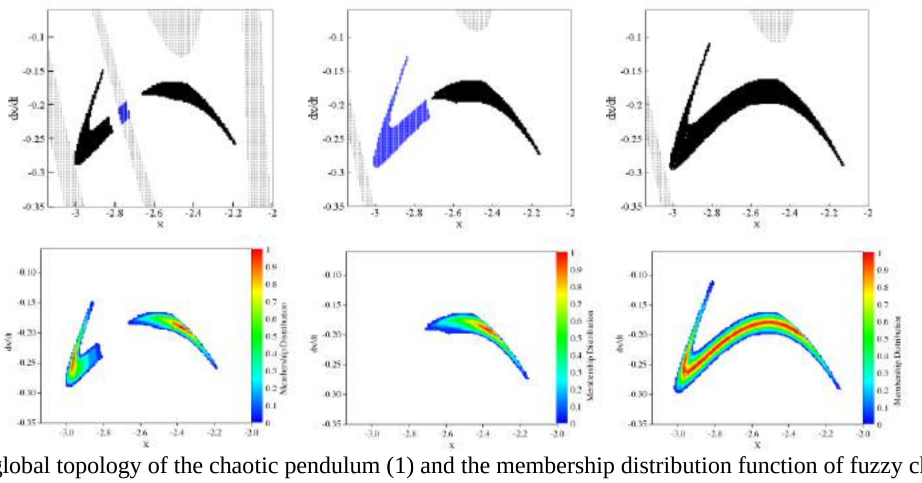

Figure 1. The global topology of the chaotic pendulum (1) and the membership distribution function of fuzzy chaotic attractors

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FORCED NONLINEAR NORMAL MODES IN ONE DISK ROTOR DYNAMICS

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ABSTRACT

A concept of nonlinear normal vibrations modes (NNMs) by Shaw and Pierre [1,2] is based on computation of invariant manifolds on which the NNMs take place. The Shaw-Pierre nonlinear normal mode is such regime when all phase coordinates are univalent functions of the selected couple of phase variables. The NNM can be determined from the system of partial derivation equations as power series. In a case of internal resonance *four* phase coordinates are active ones, and all other phase coordinates are presented as univalent functions of these active coordinates. One considers a nonlinear dynamical system under an external harmonic excitation. It is assumed that two linearized frequencies are close, and they are close to the external frequency. In this case two active generalized coordinates and two corresponding velocities are taken as independent variables to construct forced NNMs [3]. In correspondence with a principal idea of the Rauscher method, after some transformations, the n-DOF "pseudo-autonomous system" is obtained instead of the initial non-autonomous system. In the autonomous system the NNMs can be constructed. The iteration process permits to reach a necessary exactness.

It is known that different nonlinear effects must be taken into account in analysis of the rotor systems dynamics [4-6]. The forced NNMs are constructed here is the rotor systems having the internal resonance. An asymmetrical disposition of the disk in the shaft, gyroscopic effects and nonlinear flexible base are taken into account. Eight nonlinear ODEs describe displacements and rotations of the disc, and displacements in supports.

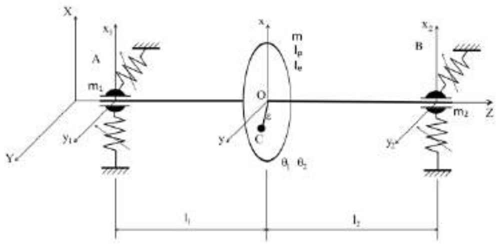

Fig.1. Principal model of the rotor dynamics.

Forced NNMs of the system are obtained. Trajectories of the NNM in configuration space are shown in Fig. 2 for some values of the system parameters. The frequency responses are obtained too. Stability and bifurcations of the resonance modes are analyzed. Numerical simulation shows a good exactness of the analytical results.

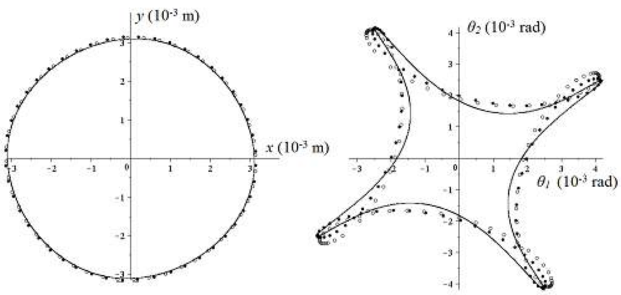

Fig. 8. Trajectories of the resonance motions in configuration space. Points and circles denote results, which are obtained by harmonic balance and NNM methods, respectively. Results of numerical simulations are shown by solid lines.

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CONSTRUCTING SIMPLE CHAOTIC SYSTEMS WITH AN ARBITRARY NUMBER OF EQUILIBIA OR OF SCROLLS

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ABSTRACT

In a typical 3D smooth autonomous chaotic system, such as the Lorenz and Rössler systems, the number of equilibria is three or less and the number of scrolls in their attractors is two or less. Today, we are able to construct a relatively simple smooth 3D autonomous chaotic system that can have any desired number of equilibria or any desired number of scrolls in its chaotic attractor.

Nowadays it is known that a 3D quadratic autonomous chaotic system can have no equilibrium, one equilibrium, two equilibria, or three equilibria. Starting with a chaotic system with only one stable equilibrium, by adding symmetry to it via a suitable local diffeomorphism, we are able to transform it to a locally topologically equivalent chaotic system with an arbitrary number of equilibria. In so doing, the stability of the equilibria can also be easily adjusted by tuning a single parameter.

Another interesting issue of constructing a 3D smooth autonomous chaotic system with an arbitrary number of scrolls is discussed next. To do so, we first establish a basic system that satisfies Shilnikov's inequalities. We then search for a heteroclinic orbit that connects the two equilibria of the basic system. Finally, we use a "copy and lift" technique and a switching control method to timely switch the dynamics between nearby sub-systems, thereby generating a chaotic attractor with multiple scrolls. Not only the number but also the positions of the scrolls in the chaotic attractor can be determined by our design method.

This talk will briefly introduce the ideas and methodologies.

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FORWARD AND BACKWARD MOTION CONTROL OF A VIBRO-IMPACT CAPSULE SYSTEM

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ABSTRACT

This paper studies the forward and backward motion control of a vibro-impact capsule system which is shown in Fig. 1. The system consists of a capsule main body interacting with an internal harmonically driven mass in the presence of dry friction, and its rectilinear motion is bidirectional. The merit of such a system is its simplicity in mechanical design and control which allows it to move independently in a complex environment. The dynamic responses of this system were studied in [1, 2], and it was shown that small variations in friction or system parameter (e.g. mass ratio) may lead to qualitative change of the dynamics. In the current work we employ a position feedback control and a velocity feedback control laws to control the capsule moving along a desired direction. We also introduce two factors to evaluate the stability of the system and the efficiency of the proposed control laws.

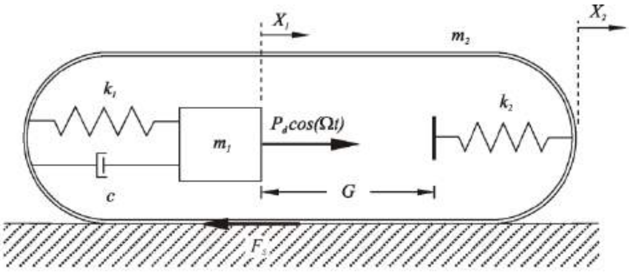

Figure 1: Physical model of the vibro-impact capsule system.

Applying the position feedback control, we are able to move the capsule system forward or backward depending on the variation of its position control gain. Our bifurcation studies show that the system response changes from period-one motion with one impact to period-one with two impacts when the value of the position control gain varies from negative to positive, and the capsule direction of motion changes from backward to forward motion. It was also revealed that the larger of the control gain, the faster of the average progression of the capsule.

To evaluate the stability of the system, we study the ratio between the variation of a system parameter and the variation of the average progression of the capsule. To evaluate the efficiency of the control law, we also introduce the ratio between the variation of the average progression and the variation of the control gain. Our numerical analysis indicates that the capsule system with larger stiffness ratio is more robust to mass variation than the one with smaller stiffness ratio, while the control law is more effective to the system with smaller stiffness ratio.

The studies of the velocity feedback control law show that the capsule moves from backward to forward when the value of the velocity control varies from negative to positive. By comparing the control signals of the feedback control laws, we obtain that the required amplitude of the velocity feedback control is much larger than the amplitude of the position feedback control which indicates that the velocity one needs more energy. To evaluate the energy consumption, we use the ratio of the capsule progression per period of the external excitation to the work done by the external force in one period. By calculating this ratio, we find that the velocity control is more efficient than the position control, and the larger of the velocity control gain, the much more efficient it is.

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MONITORING OF THE CHARACTERISTIC PARAMETER CHANGES OF A NONLINEAR OSCILLATOR BY NONLINEAR SYSTEM MODELING AND ANALYSIS

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ABSTRACT

Impact drilling systems can be found in many engineering applications [1]. Traditionally, these systems can be approximated by oscillators with a piecewise linear nonlinear stiffness. From physical point of view, the stiffness change can be related to different material densities found by the probe, an information that can be used for monitoring the drilling process so as to more effectively control the drill speed. Therefore, it is important to develop methods that can effectively identify the stiffness changes from measurable signals in order to improve the control of the drilling process.
The behavior of piecewise linear oscillators subject to sinusoidal inputs have been extensively studied [1]. However, most of the investigations have been focused on characterizing complex dynamical regimes observed when the input amplitude or frequency is changed by means of bifurcation maps [2]. Although this information provides a useful insight about the drill response when working on a known environment, it cannot be used to identify a new scenario, which is associated with a physically meaningful stiffness change. In this study we address this issue by using nonlinear systems modeling and analysis.
The methodology consists of building a polynomial NARMAX (Nonlinear Auto-Regressive Moving Average with eXogenous inputs) model [3] using input-output data from an impact oscillator system. Then, the response of the identified model is analyzed in the frequency domain by extracting the system NOFRFs (Nonlinear Output Frequency Response Functions) [4] over a certain frequency range of interest, using a novel general framework based on ALEs (Associated Linear Equations) [5]. Finally, a simple NOFRFs based index is used to distinguish different scenarios of stiffness for the purpose of monitoring the oscillator's operational conditions.
Simulation studies have been conducted where the methodology was applied to a nonlinear oscillator described by a non-dimensional equation of motion of the form

$$\frac{d^2x(t)}{dt^2} + 2\xi\frac{dx(t)}{dt} + x(t) + \beta(x(t)-e)h(x(t)-e) = u(t) \quad (1)$$

In (1), $x(t)$ is the displacement of motion, t is the time, ξ is the damping ratio, β is the stiffness ratio, e represents a gap, and $u(t)$ is the external excitation. The system was first identified from a sinusoidal input $u(t) = \Gamma\, sin(\omega t)$, and corresponding response of model (1). Then, the identified model was analyzed in the frequency domain by extracting the system's NOFRFs and evaluating a NOFRFs based index as defined by

$$M_n = \frac{1}{N}\sum_{k=1}^{N}|G_n(k)|^2 \quad (2)$$

In (2), $G_n(k)$ is the n-th order NOFRF at frequency component k, and N is the total number of frequency components of interest. M_1, M_2, and M_3 obtained with $N=1000$ are shown in Table 1. The results demonstrate that the simple NOFRFs based index can significantly distinguish different scenarios of stiffness changes and can, therefore, be used to monitor the nonlinear oscillator system's working conditions.

β	M_1	M_2	M_3
1	2.9092	0.0359	0.0002
6	2.7188	2.0404	49.9415
10	0.6224	0.2011	26.6961
20	8.8768	28.6581	266.0859

Table 1 Values of the NOFRF index for different stiffness ratio β

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HOW COMMON PERIODIC STABLE BEHAVIOR APPEARS IN NONLINEAR DISSIPATIVE (MECHANICAL) SYSTEMS

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ABSTRACT

The emergence of regular behaviour is one of the most studied topics in nonlinear dynamical systems. It is known that by the changing of an accessible parameter of a chaotic system, chaos can be replaced by a stable periodic behaviour. In this talk, I will review some recent results which accounts to clarifying what are the general conditions under which one can replace chaos into stable periodic behaviour (or vice-versa) by a parameter alteration.

I will start by explaining the windows conjecture [1,2] that describes why for systems that possess k positive Lyapunov exponents, it is possible to find stable periodic behaviour by altering simultaneously k control parameters. Then, I will discuss some recent experiments to demonstrate these results in nonlinear electronic circuits [3,4], discuss recent works that show that periodic windows can be used to understand the behaviour of a different class of systems, varying from mechanical systems [5] to bird population dynamics [6], and discuss other works such as in Refs. [6,7] that clarify the relationship between the structure of periodic windows and homoclinic bifurcations.

Finally, I will present preliminary results of a work being carried out at the University of Aberdeen about the relationship between the structure of these periodic windows, homoclinic bifurcations, and the existence of parameters for which the lateral deviation of railway vehicles changes abruptly. We investigate the parameter dependence of the lateral dynamics described by a rail wheelset model [8], for instance, as shown in Fig. 1, the lateral deviation suddenly changes for certain values of the train speed.

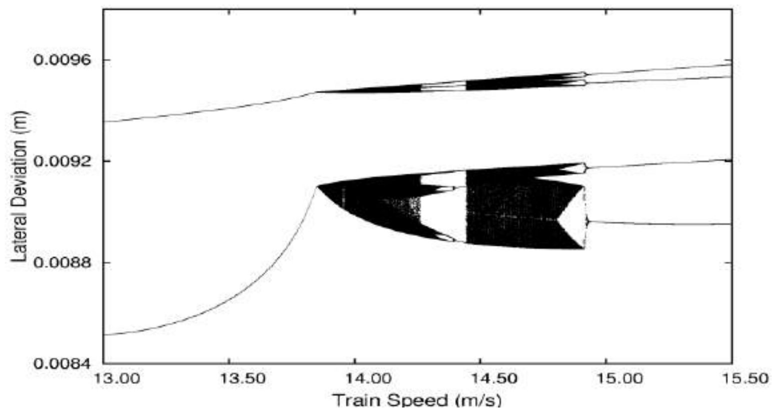

Fig. 1: Transitions similar to those observed in Fig. 1 must be also verified for intervals of others system parameters settled by the considered wheelset model.

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ONE-DIMENSIONAL CHAOS IN A SYSTEM WITH DRY FRICTION

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ABSTRACT

We offer a new method which can be used to find chaotic dynamics in systems with a dry friction. This method is based on reduction to a low-dimensional dynamics [2,3] (see also [4] for another approach) and application of criteria of chaos for mappings of segments [1, Part 3]. To illustrate this method, we consider a simple system, consisting of a mass, moving under the action of a harmonic force $F(t) = a \sin t + b$ and a weightless slider, providing a dry friction with a maximal value q. Suppose that

$$a > 0, \qquad b > 0, \qquad q \in (a \sin \vartheta_0 , a) \tag{1}$$

Here $\vartheta_0 \approx 0.81047$ is the zero of the equation $\pi - \theta = \cot(\theta/2)$, considered on $(0, \pi/2)$. Let x be the position of the mass, y be the position of the delimiter and t be the variable of time. The following table illustrates possible regimes of the considered system. The first three regimes can be observed on open intervals of time. The last two regimes are instantaneous and correspond to transitions between principal regimes.

Type of motion.	**Conditions**	**Equations of motion.**
Free motion **(f)**.	$x < y$	$\ddot{x} = F(t);\quad \dot{y} = 0$ (2)
Progression **(p)**	$x = y;\ \dot{x} > 0$	$\ddot{x} = F(t) - q;\quad y = x$ (3)
Stop **(s)**	$x = y; \dot{x} = 0; F(t) \in (0, q)$	$x = y, \dot{x} = \dot{y} = 0.$ (4)
Instantaneous stop **(is)**	$x = y; \dot{x} = 0; F(t) \notin (0, q)$	Not applicable.
Free flight to progression transition **(fp)**	$x = y;\ \dot{x} > 0$ for a fixed instant $\bar{t}$ and $x < y$ in a left neighborhood of $\bar{t}$	Not applicable.

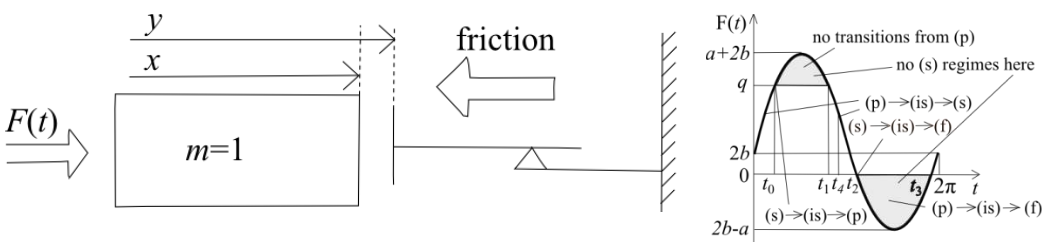

Figure 1: The considered dynamical system and corresponding regimes.

Let $t_{2,3} \in [0, 2\pi)$ be zeros of the function $F(t) = a \sin t + b$ (Figure 1). The following theorem demonstrates that a chaotic behavior can be observed in the considered system provided b is small.

Theorem. *For all a and q, satisfying inequalities* (1) *there exists a $b_0 = b_0(a, q) > 0$ such that for all $b \in (0, b_0)$ the mechanical system, described by equations* (2), (3) *and* (4) *is chaotic in the following sense. The phase of transition to free flight of the motion uniquely defines the phase of the next transition. This defines a discontinuous mapping T from the segment $[t_2, t_3)$ into itself. There exist two disjoint segments J_0 and J_1 of the segment $[t_2, 3\pi/2]$ such that $[t\_2, 3\pi/2] \subset T(J_i)$* $(i = 0,1)$ *and T is continuous on both segments J_i. Particularly, there exists an infinite set P of periodic points of the mapping T. Minimal periods of points of P are unbounded.*

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ADVANCES IN USING THE STATIC-DYNAMIC ANALOGY IN APPLIED MECHANICS

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ABSTRACT

Using various twisted rod models as a means of illustration, we demonstrate the new features of unstable shell-like buckling that have been uncovered in recent years by the skilful use of the static-dynamic analogy. These include the manner in which localization of the post-buckling response is governed by the equal-energy Maxwell criterion.

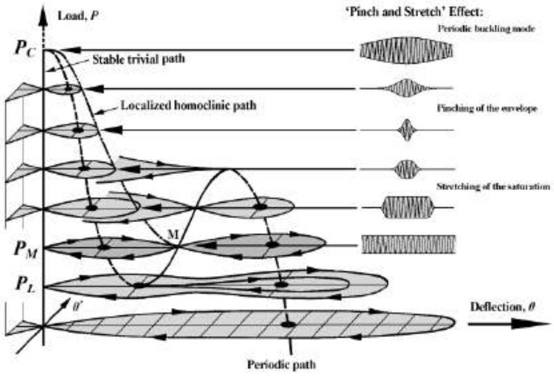

Figure 1: A composite schematic illustrating the localization of the buckle pattern, applicable to shell-like instabilities in structures and Turing-type pattern formation in chemical and biological kinetics.

A useful archetypal model for studying localization phenomena is the long (essentially infinite) spatially deforming twisted elastic rod subjected to an end tension. This was first studied theoretically and experimentally by Thompson and Champneys [1] in 1996. The isotropic (anisotropic) twisted rod is mathematically equivalent to the symmetric (non-symmetric) spinning top if we identify the axial coordinate of the rod with the time variable of the top. A comprehensive survey for an isotropic rod (with equal bending stiffness about any axis) is given by van der Heijden and Thompson [2]. Unlike the anisotropic rod which can generate spatial chaos, this is an integrable system whose response is easily understood via an equivalent oscillator. Under the variation of a loading parameter, P, non-trivial fixed points of the oscillator define a sub-critical periodic path, while saddle connections define a co-existing localizing path, each bifurcating from the trivial un-buckled solution at P_C. Both paths carry a continuously falling load, and so exhibit no lower buckling load P_L or Maxwell load P_M. A second model that we can draw on is the long twisted isotropic rod constrained to lie in a cylinder which was studied by van der Heijden [3] in 2001. Its generalization to anisotropic rods [4] involves spatially chaotic deformations, but this latter paper is also a useful reference for the isotropic rod. In particular, Fig. 14 shows for the isotropic rod the stabilization of the periodic post-buckling path at P_L. The falling localizing equilibrium path is then destroyed on colliding with the stabilized path at the Maxwell load P_M.

For the free twisted rod we have a finite P_C but no P_L or P_M while for the rod in a cylinder we have a finite P_L and P_M but P_C is infinite. The schematic composite diagram of Fig 1 illustrates a general case, drawing on these two models, in which all three critical loads are finite. In this paper we use this figure to outline many of the recent advances, including in particular those of practical relevance in solids and fluids.

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