

# Numerics and Theory for Stochastic Evolution Equations

SFB 701

Nov. 22nd - 24th, 2006

Department of Mathematics, Bielefeld University

The aim of the workshop is to review the recent trends on stochastic evolution equations focusing on the constructive and computational aspects. The invited main speakers are leading experts in their fields and will present an overview of their recent research, directed also towards non-specialists and those entering the field.

## Organizers

- Wolf-Jürgen Beyn ([beyn@math.uni-bielefeld.de](mailto:beyn@math.uni-bielefeld.de))
- Michael Röckner ([roeckner@math.uni-bielefeld.de](mailto:roeckner@math.uni-bielefeld.de))
- Christian Rohde ([Christian.Rohde@mathematik.uni-stuttgart.de](mailto:Christian.Rohde@mathematik.uni-stuttgart.de))

## Speakers

Annunziato, Mario (Salerno)  
Buckwar, Evelyn (HU Berlin)  
Dirr, Nicolas (MPI Leipzig)  
Gentz, Barbara (Bielefeld)  
Grecksch, Wilfried (Halle)  
Grün, Günther (Erlangen)  
Higham, Des (Strathclyde)  
Imkeller, Peter (HU Berlin)  
Kloeden, Peter (Frankfurt)  
Kuksin Sergei (Edinburgh)  
Larsson, Stig (Göteborg)  
Lord, Gabriel (Edinburgh)

Loy, Matthias (Tübingen)  
Maier-Paape, Stanislaus (Aachen)  
Malham, Simon (Edinburgh)  
Rascanu, Aurel (Iasi)  
Ritter, Klaus (Darmstadt)  
Schmalfuß, Björn (Paderborn)  
Shardlow, Tony (Manchester)  
Sickenberger, Thorsten (HU Berlin)  
Stannat, Wilhelm (Darmstadt)  
Stuart, Andrew (Warwick)  
Zouraris, Georgios (Kreta)

[www.math.uni-bielefeld.de/~beyn/workshop2006/index.html](http://www.math.uni-bielefeld.de/~beyn/workshop2006/index.html)

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The workshop is part of the conference program of the DFG-sponsored CRC 701 “Spectral Structures and Topological Methods in Mathematics” at Bielefeld University.



## Conference Room: A3-126

### Program

#### Wednesday, 22nd

- 08:45 **Registration**
- 09:00 **Welcome**
- 09:05 **Imkeller, Peter (HU Berlin)**  
Meta-stability in stochastic differential equations induced by Levy noise
- 09:50 **Shardlow, Tony (Manchester)**  
Stochastic PDEs and excitable media
- 10:35 **Coffee break**
- 11:05 **Maier-Paape, Stanislaus (Aachen)**  
Phase Separation in Stochastic Cahn-Hilliard Models
- 11:50 **Zouraris, Georgios (Kreta)**  
Finite element methods for a fourth order stochastic parabolic equation
- 12:15 **Lunch break**
- 14:00 **Larsson, Stig (Göteborg)**  
Finite element approximation of parabolic stochastic PDEs
- 14:45 **Schmalfuß, Björn (Paderborn)**  
Dynamics for numerical schemes for SPDE
- 15:30 **Coffee break**
- 16:00 **Rascanu, Aurel (Iasi)**  
Parabolic backward stochastic variational inequalities
- 16:45 **Dirr, Nicolas (MPI Leipzig)**  
Interfaces in heterogeneous environment
- 17:30 **Sickenberger, Thorsten (HU Berlin)**  
Adaptive Methods for SDAEs with small noise
- 17:55 **Finish**

#### Thursday, 23rd

- 09:00 **Kloeden, Peter (Frankfurt)**  
The pathwise numerical approximation of stationary solutions of semilinear stochastic evolution
- 09:45 **Grecksch, Wilfried (Halle)**  
An infinite-dimensional fractional linear-quadratic regulator problem

- 10:30 **Coffee break**
- 11:00 **Ritter, Klaus (Darmstadt)**  
Optimal Approximation for a Class of Stochastic Heat Equations
- 11:45 **Buckwar, Evelyn (HU Berlin)**  
Weak convergence of the Euler-Maruyama scheme for stochastic delay differential equations
- 12:10 **Lunch break**
- 14:00 **Stuart, Andrew (Warwick)**  
MCMC Methods for Sampling Conditioned Diffusions
- 14:45 **Gentz, Barbara (Bielefeld)**  
Desynchronisation of coupled bistable oscillators perturbed by additive white noise
- 15:30 **Coffee break**
- 16:00 **Malham, Simon (Edinburgh)**  
Stochastic Lie group integrators
- 16:45 **Annunziato, Mario (Salerno)**  
On a finite difference method for piecewise deterministic processes with memory and its parallel algorithm implementation
- 17:10 **Loy, Matthias (Tübingen)**  
Exponential integrators for the linear stochastic Schroedinger equation
- 17:35 **Finish**
- 19:00 **Brauhaus (Tavern)**

## Friday, 24th

- 09:00 **Kuksin Sergei (Edinburgh)**  
Statistical hydrodynamics in thin 3D layers
- 09:45 **Higham, Des (Strathclyde)**  
Multi-Level Monte Carlo
- 10:30 **Coffee break**
- 11:00 **Lord, Gabriel (Edinburgh)**  
An exponential type scheme for stochastic PDE
- 11:45 **Grün, Günther (Erlangen)**  
Thin-film flow influenced by thermal fluctuations
- 12:10 **Stannat, Wilhelm (Darmstadt)**  
Particle filters and the Kushner-Stratonovitch equation
- 12:55 **End of the conference**

## Abstracts

### On a finite difference method for piecewise deterministic processes with memory and its parallel algorithm implementation

**Annunziato, Mario (Salerno)**

We treat the problem of finding the distribution function for a class of piecewise deterministic processes with memory, by numerically solving the associated Liouville-Master Equation. That equation is composed of a linear system of hyperbolic PDEs and of boundary conditions that depend on the integral over the interior of the solution domain.

We illustrate a numerical scheme of the first order, resulting by a combination of the upwind scheme and of a direct quadrature. We show a Courant-Friedrichs-Lewy condition that both ensures convergence and the monotonic property (positivity) of the numerical solution. We describe the parallel algorithm, implemented by using the MPI library, and the results of the performance tests for a known problem.

### Weak convergence of the Euler-Maruyama scheme for stochastic delay differential equations

**Buckwar, Evelyn (HU Berlin)**

We consider weak approximations of solutions of stochastic delay differential equations with discrete delays. We discuss the Euler-Maruyama scheme for this type of equations and show that it has order of weak convergence 1, as in the case of stochastic ordinary differential equations, i.e., equations without delay. Although the set-up is non-anticipating, our approach uses the Malliavin calculus and the anticipating stochastic analysis techniques of Nualart and Pardoux. Numerical experiments illustrate the theoretical findings. The results presented are joint work with Rachel Kuske, Salah Mohammed and Tony Shardlow.

### Interfaces in heterogeneous environment

**Dirr, Nicolas (MPI Leipzig)**

We describe results concerning qualitative properties of equations that model interfaces (e.g. phase boundaries) in periodic or random media. The evolution is governed by a gradient flow for the interfacial energy which is perturbed by stochastic or periodic terms (heterogeneities) on a small scale. Aspects of the effective behavior on the larger scale are derived.

# Desynchronisation of coupled bistable oscillators perturbed by additive white noise

**Gentz, Barbara (Bielefeld)**

We consider a chain of  $N$  overdamped bistable oscillators with nearest-neighbour coupling. Each site is perturbed by an independent white noise, modeling the influence of a heat reservoir. This system is described by a set of stochastic differential equations on  $\mathbb{R}^{\mathbb{Z}/N\mathbb{Z}}$ .

For sufficiently large coupling, synchronisation is observed. We show that, as the coupling decreases, the system desynchronises in a sequence of symmetry-breaking bifurcations, corresponding to a gradual increase in the number of local minima of the potential landscape. Large-deviation techniques provide estimates on activation energy as well as on optimal transition paths and times from one synchronised state to other.

Joint work with Nils Berglund (CPT, Marseille) and Bastien Fernandez (CPT, Marseille).

# An infinite-dimensional fractional linear-quadratic regulator problem

**Grecksch, Wilfried (Halle)**

We consider a fractional analogue of the infinite-dimensional linear quadratic regulator problem. The state process is governed by a linear stochastic evolution equation with additive fractional Brownian noise. The state process is completely observed. The admissible controls are closed-loop controls. The optimal control is constructed explicitly which minimizes a quadratic goal functional. Adjoint equations and the so called Gaussian fundamental martingale are applied.

# Thin-film flow influenced by thermal fluctuations

**Grün, Günther (Erlangen)**

We will be concerned with the effects thermal fluctuations have on thin-film (de)wetting. Starting from incompressible Navier-Stokes equations with noise, we use long-wave approximation and Fokker-Planck-type arguments to derive a fourth-order degenerate parabolic stochastic partial differential equation – the stochastic thin-film equation. We propose a discretization scheme and give both formal and numerical evidence for our conjecture that thermal fluctuations may resolve discrepancies with respect to time-scales of dewetting which have been observed recently in comparing physical experiments and deterministic numerical simulations. This is joint work with K. Mecke and M. Rauscher.

# Multi-Level Monte Carlo

## Higham, Des (Strathclyde)

Many problems require the expected value of a functional of the solution to a stochastic differential equation (SDE). A notable application area is option valuation in mathematical finance. A simple and widely used computational approach is to simulate the SDE numerically, for example, with the Euler-Maruyama method, and to regard each approximate solution path as a sample in a Monte Carlo algorithm. In this case there are two sources of error: (1) Discretization: Euler-Maruyama does not follow paths exactly, and (2) Statistical: the sample mean does not match the true expected value. By balancing the contribution of these two types of error we obtain an algorithm that requires  $\mathcal{O}(\epsilon^{-3})$  effort to achieve an accuracy of  $\mathcal{O}(\epsilon)$ .

Recently, in the report "Multi-level Monte Carlo path simulation", Report NA-06/03, Oxford University Computing Laboratory, 2006, Mike Giles showed the remarkable result that the computational effort can be reduced to  $\mathcal{O}(\epsilon^{-2}(\log \epsilon)^2)$ . This new 'multi-level' Monte Carlo algorithm uses an approach that bears some resemblance to multigrid methods and more generally to Richardson extrapolation. Surprisingly, both the analysis and the actual performance of the algorithm depend heavily on the property of strong convergence.

After describing Giles' algorithm, we will give some new strong convergence results that justify the use of the algorithm for various types of financial option valuation. This is joint work with Mike Giles and Xuerong Mao.

# Meta-stability in stochastic differential equations induced by Levy noise

## Imkeller, Peter (HU Berlin)

A spectral analysis of the time series representing average temperatures during the last ice age featuring the Dansgaard-Oeschger events reveals an  $\alpha$ -stable noise component with an  $\alpha \sim 1.78$ . Based on this observation, papers in the physics literature attempted a qualitative interpretation by studying diffusion equations that describe simple dynamical systems perturbed by small Lévy noise. We study exit and transition problems for solutions of stochastic differential equations of this type. Due to the heavy-tail nature of the  $\alpha$ -stable component of the noise, the results differ strongly from the well known case of purely Gaussian perturbations.

# The pathwise numerical approximation of stationary solutions of semilinear stochastic evolution

**Kloeden, Peter (Frankfurt)**

Under a one-sided dissipative Lipschitz condition on its drift, a stochastic evolution equation with additive noise of the reaction-diffusion type is shown to have a unique stochastic stationary solution which pathwise attracts all other solutions. A similar situation holds for each Galerkin approximation and each implicit Euler scheme applied to these Galerkin approximations. Moreover, the stationary solution of the Euler schemes converges pathwise to that of the Galerkin system as the stepsize tends to zero and the stationary solutions of the Galerkin systems converge pathwise to that of the evolution equation as the dimension increases. The analysis is carried out on random partial and ordinary differential equations obtained from their stochastic counterparts by subtraction of appropriate Ornstein-Uhlenbeck stationary solutions.

# Statistical hydrodynamics in thin 3D layers

**Kuksin, Sergei (Edinburgh)**

I will consider the Navier-Stokes equation in an  $\epsilon$ -thin 3D layer. The equation is perturbed by a non-degenerate random force. I will show that, firstly, when  $\epsilon \ll 1$  the equation has a unique stationary measure which describes asymptotic in time statistical properties of solutions and, secondly, after the averaging in the thin direction this measure converges (as  $\epsilon$  goes to zero) to a unique stationary measure for the Navier-Stokes equation on the corresponding 2D surface. Thus, this 2D equation describes asymptotic in time and limiting in  $\epsilon$  statistical properties of the Navier-Stokes flow in the narrow 3D domain.

# Finite element approximation of parabolic stochastic PDEs

**Larsson, Stig (Göteborg)**

We consider a linear parabolic stochastic differential equation in several spatial variables forced by additive colored space-time noise. The equations are discretized in the spatial variable by a finite element method. In order to capture the multiscale behavior of the colored noise we use a hierarchical wavelet basis for the finite element space. We prove strong convergence estimates.

# **An exponential type scheme for stochastic PDE**

**Lord, Gabriel (Edinburgh)**

We discretize a stochastic parabolic PDE using an exponential type integrator in time. In space we use a Galerkin approximation and investigate how many modes are required for an efficient scheme.

# **Exponential integrators for the linear stochastic Schrödinger equation**

**Loy, Matthias (Tübingen)**

Certain Magnus integrators and Trotter splittings are applied to the (spatial) pseudo-spectral discretized linear stochastic Schrödinger equation. Under the assumption of commutator- and energy-bounds strong error behaviour for the presented methods are discussed. Further, it is shown that the Trotter splitting methods are in general preferable.

# **Phase Separation in Stochastic Cahn-Hilliard Models**

**Maier-Paape, Stanislaus (Aachen)**

The Cahn-Hilliard equation is one of the fundamental models for phase separation dynamics in metal alloys. On a qualitative level, it can successfully describe phenomena such as spinodal decomposition and nucleation. Yet, as deterministic partial differential equation it does not account for thermal fluctuations or similar random effects. In this survey talk we describe some dynamical aspects of a stochastic version of the model due to Cook. These include recent results on spinodal decomposition, as well as a brief discussion of nucleation and its relation to the deterministic attractor structure. In addition, differences between the deterministic and the stochastic dynamics are discussed.



# Optimal Approximation for a Class of Stochastic Heat Equations

**Ritter, Klaus (Darmstadt)**

We study numerical approximation of stochastic heat equations

$$dX(t) = \Delta X(t) dt + B(t, X(t)) dW(t)$$

driven by nuclear or by space-time white noise on the spatial domain  $]0, 1[^d$ . The error of an approximation is defined in  $L_2$ -sense, and the computational cost of an algorithm is measured by the number of evaluations of the one-dimensional components of  $W$ .

We are interested in the following basic question: what is the minimal error  $e(n)$  that can be achieved by any algorithm with cost at most  $n$ . The sequence of minimal errors quantifies the intrinsic difficulty of our computational problem.

We determine the asymptotic behaviour of  $e(n)$  and we present almost optimal algorithms, i.e, algorithms with error close to  $e(n)$  and cost bounded by  $n$ . In the case of nuclear noise it is crucial to use a non-uniform time discretization in order to achieve optimality.

## Dynamics for numerical schemes for SPDE

**Schmalfuß, Björn (Paderborn)**

We introduce a numerical scheme for a SPDE with additive white noise. This numerical scheme defines a random dynamical system. To investigate the long-time behavior of such a numerical scheme the term pullback attractor is introduced. The existence of this type of attractor for the numerical scheme is proved and relations to the attractor of the original spde are discussed.

## Stochastic PDEs and excitable media

**Shardlow, Tony (Manchester)**

I will discuss some stochastic PDEs that arise in modelling excitable media. Two effects will be discussed: nucleation of waves occurs due to background noise and we show how the type of wave that is nucleated depends on system parameters. Waves are found to propagate in a larger region of parameter space when noise is present: we investigate numerically.

# Adaptive Methods for SDAEs with small noise

**Sickenberger, Thorsten (HU Berlin)**

In this talk the numerical approximation of solutions of Ito stochastic differential algebraic equations (SDAEs) with small noise is considered. SDAEs arise from transient noise analysis in circuit simulation.

We discuss adaptive linear multi-step methods and their mean-square convergence. For the case of small noise we present a strategy for controlling the step-size. It is based on estimating the mean-square local errors by an ensemble of solution paths that are computed simultaneously. This leads to step-size sequences that are identical for all paths.

Test results illustrate the performance of the presented methods. (joint work with Renate Winkler)

# Particle filters and the Kushner-Stratonovitch equation

**Stannat, Wilhelm (Darmstadt)**

The Kushner-Stratonovitch equation is a measure-valued stochastic partial differential equation arising in stochastic filtering theory and describing the conditional distribution of a Markovian signal observed through additive noise. The solution of the Kushner-Stratonovitch equation can be approximated by weighted particle systems (called particle filters in this context) with a mutation/selection mechanism. Using a variational approach, we discuss two theoretical issues concerning the mean squared error of the approximation and demonstrate the use of a ground state transform to increase its efficiency.

# MCMC Methods for Sampling Conditioned Diffusions

**Stuart, Andrew (Warwick)**

There are a wide variety of applications which can be cast as sampling problems for conditioned SDEs (diffusion processes). Examples include nonlinear filtering in signal processing, data assimilation in the ocean/atmosphere sciences, data interpolation in econometrics, and finding transition pathways in molecular systems. In all these examples the object to sample is a path in time, and is hence infinite dimensional. We describe abstract MCMC methods for sampling such problems, based on writing the target measure as a change of measure from a Gaussian process. We generalize the independence sampler, Metropolis adjusted Langevin algorithm and Random Walk Metropolis algorithm to this path space setting. We give an overview of the subject area, describing the analytical, statistical and computational challenges, and illustrating applicability of the techniques being developed.

Collaboration with:

Alex Beskos, Martin Hairer, Jochen Voss (Warwick), David White (Warwick), Gareth Roberts (Lancaster)

# Finite element methods for a fourth order stochastic parabolic equation

**Zouraris, Georgios (Kreta)**

We consider, as the main problem, an initial- and Dirichlet boundary- value problem for a fourth-order linear stochastic parabolic equation, in one space dimension, forced by an additive space-time white noise.

Since the solution of the problem is not regular, following [Allen, Novosel and Zhang, Stochastics Stochastics Rep., vol.64, 1998], we replace the space-time white noise by a properly chosen discrete space-time white noise kernel to get a modified fourth-order linear stochastic parabolic problem.

Fully-discrete approximations to the solution of the modified problem are constructed by using, for discretization in space, a standard Galerkin finite element method and, for time-stepping, the Backward Euler method.

We derive a priori estimates

(i) for the difference between the solution of the main problem and the solution of the modified problem, and

(ii) for the numerical method approximation error to the solution of the modified problem.

## Participants

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