An optimisation approach to establish dynamical equivalence for soft and rigid impact models

Bo Tian[a], Shan Yin[a], Joseph Páez Chávez[b,c] and Yang Liu[a,*]
[a] *Exeter Small-Scale Robotics Laboratory, University of Exeter, Exeter, UK, EX4 4QF*
[b] *Center for Applied Dynamical Systems and Computational Methods (CADSCOM), Faculty of Natural Sciences and Mathematics, Escuela Superior Politécnica del Litoral, Guayaquil, Ecuador*
[c] *Center for Dynamics, Department of Mathematics, TU Dresden, D-01062 Dresden, Germany*

Summary. This work studies a computational approach aimed at establishing equivalent dynamical responses within oscillatory impacting systems subject to soft and rigid constraints. The proposed method incorporates an adaptive differential evolution algorithm with the Metropolis criterion to determine the stiffness and damping parameters of the soft constraint for a prescribed coefficient of restitution governing the rigid constraint. This algorithm aims to achieve equal energy dissipation between the two constraints. Upon examining the dynamical responses of the two impact cases, they exhibit nearly identical outcomes in the two-parameter bifurcation diagrams when subjected to a large restitution coefficient. However, discrepancies arise when the restitution coefficient is low. Detailed numerical tests, conducted using the proposed method, demonstrate enhanced effectiveness compared to previous techniques, such as the prediction formulae outlined by Okolewski and Blazejczyk-Okolewska (Chaos, 31:083110, 2021).

Introduction

Mechanical systems involving impacts have garnered significant attention due to their rich, complex nonlinear characteristics and extensive use across various engineering applications, e.g., percussive drilling [1] and the impact-driven robot [2]. However, selecting an appropriate model still remains challenging when modeling vibro-impact systems. Okolewski et al. [3, 4] discussed the practicalities of using a soft and a hard impact model by exploring a broader range of restitution coefficients as a measure of energy dissipation. They compared the distinctions between the two impact models while accounting for equivalent energy dissipation, and proposed a formula to determine the base's stiffness and damping. However, the formula has a notable limitation, since it necessitates specifying either the base's stiffness or damping to calculate the other parameter with a given restitution coefficient. To overcome this limitation, an adaptive Differential Evolution algorithm [5] incorporating the Metropolis criterion [6] is proposed in this work to determine the stiffness ratio (β) and damping ratio (ζ_1) of the soft constraint, corresponding to a given coefficient of restitution (r) governing the rigid constraint. The proposed approach can be applied in the experimental identification of physical properties of an impact surface. Also, it is convenient to consider rather soft models for impacting systems since some integrators cannot handle discontinuities accurately, produced by the jumps in the velocities right before and after impacts.

Mathematical description and methodology

The single-degree-of-freedom impact oscillators with a one-sided soft and rigid constraints are considered in this work. For the soft impact oscillator, the constraint takes the form of a spring-damper pair, while the rigid impact oscillator's constraint follows a coefficient of restitution law. Both impact oscillators are driven by an external harmonic excitation. For the soft impact model, its non-dimensional equations of motion can be expressed as follows:

$$\ddot{x} + 2\zeta\dot{x} + x + (\beta(x-d) + \zeta_1\dot{x})H_k(x-d) = A\sin(\omega\tau + \varphi). \tag{1}$$

The governing equation of the rigid impact model is written as follows:

$$\begin{cases} \ddot{x} + 2\zeta\dot{x} + x = A\sin(\omega\tau + \varphi), & x < d, \\ \dot{x}_+ = -r\dot{x}_-, & x = d. \end{cases} \tag{2}$$

where x and $\dot{x}$ are the non-dimensional displacement and velocity of the oscillator, respectively. τ is the dimensionless time, ω is the frequency ratio, φ is the phase shift, β is the stiffness ratio between the oscillator and the soft constraint, ζ is the damping ratio, ζ_1 is the damping ratio of the soft constraint, A is the forcing amplitude, d is the non-dimensionalised gap, and $H(\cdot)$ is the Heaviside step function. $\dot{x}_+$ represents the impact velocity immediately after the impact instant, $\dot{x}_-$ denotes the impact velocity right before the impact instant, and r is the coefficient of restitution.

The equivalency of the two impact models was considered from two aspects, i.e., the energy dissipation during the impact and a negligible contact time duration. Assuming that the two impact models start from the same initial conditions $(x, \dot{x}, \theta) = (d, \dot{x}_0, \theta_0)$ on the impact surface, then the velocity $\dot{x}_1$ of the soft impact model leaving the impact surface $x = d$ after time duration τ can be obtained from the following equation

$$\begin{cases} d = x(\beta, \zeta_1, A, \omega, d, \dot{x}_0, \varphi_0, \tau), \\ \dot{x}_1 = \dot{x}(\beta, \zeta_1, A, \omega, d, \dot{x}_0, \varphi_0, \tau), \end{cases} \tag{3}$$

where $x(*)$ and $\dot{x}(*)$ denote the solution of the soft impact model (1). Hence, the quantitative analysis indices to measure the difference in the dynamical response of the two impact models are defined as follows:

$$e_1 = \dot{x}_1 - (-r\dot{x}_0), \quad e_2 = \tau, \tag{4}$$

where $-r\dot{x}_0$ indicates the velocity of the rigid impact model immediately after the impact event. Then, a generic formula of the quantitative index can be expressed as

$$e_{g,j}(\boldsymbol{\alpha}, \boldsymbol{\beta}_j) = \sqrt{e_1^2 + e_2^2}, \tag{5}$$

where $\boldsymbol{\alpha} = (r, \beta, \zeta_1)$ and $\boldsymbol{\beta}_j = (A_j, \omega_j, \dot{x}_{0,j}, \theta_{0,j})$. With given $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}_j$, the quantitative index $e_{g,j}$ can be calculated by long-time integration of Eqs. (1) and (2). To measure the global difference in the dynamical response of the two impact models for any $\boldsymbol{\alpha} = (r, \beta, \zeta_1)$, a statistical indicator is introduced below

$$e_g(\boldsymbol{\alpha}) = \sqrt{\frac{1}{N}\sum_{j=1}^{N} e_{g,j}^2(\boldsymbol{\alpha}, \boldsymbol{\beta}_j)}, \tag{6}$$

By sampling $\boldsymbol{\beta}$, the best parameter set $\boldsymbol{\alpha} = (\beta, \zeta_1)$ can be determined by minimising $e_g(\boldsymbol{\alpha})$ with any given r by using an adaptive Differential Evolution algorithm with Metropolis criterion.

Results analysis

A detailed comparison of the two-parameter dynamical response of the rigid and soft impact oscillators are discussed here, utilising the optimal contact parameter set (r, β, ζ_1) from the parameter optimisation. Under a large $r = 0.85$, comparisons between two-parameter dynamic response of the two cases as depicted in Fig. 1(a) and (b) reveal remarkable similarities in the dynamics of the two impact oscillators. For instance, in the upper-left corner of the (ω, A) parameter plane, both oscillators consistently exhibit transitions from 1/3 motion to 1/2 motion. Similarly, in the upper-right corner, responses switches from 4/3 motion to 4/2 motion, then to 2/1 motion. The match rate between these two oscillators, as depicted in Fig. 1(c), is 91.9%.

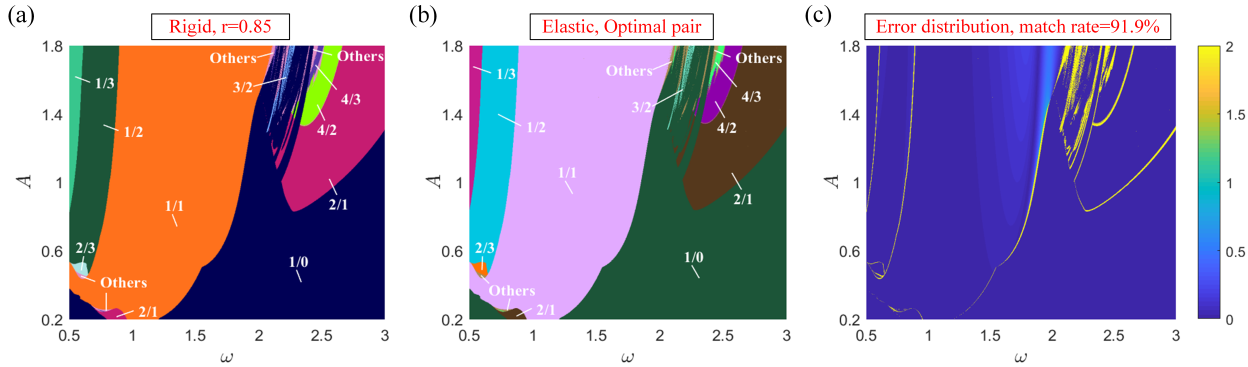

Figure 1: Comparisons between the two-parameter dynamic responses of the two impact models under the optimal contact parameter set $(r, \beta, \zeta_1) = (0.85, 10^4, 10.3288336)$. (a) The two-parameter dynamics of the rigid impact case with $r = 0.85$. (b) The two-parameter dynamics of the soft impact case with $(\beta, \zeta_1) = (10^4, 10.3288336)$. (c) The error distribution diagram of the two impact cases with an error e_c defined by the special Poincaré sections, where the match rate indicates where $e_c < 0.1$.

Conclusions

The optimisation approach studied in this work overcomes the limitations in impact modeling methods outlined in Ref. [4], and succeeds optimising optimal contact parameter pairs (β, ζ_1) with any given r, producing a remarkable quantitative and qualitative similarity in the dynamical responses of the considered impact oscillators. For example, when the restitution coefficient is high, the soft and rigid impact models show matching results in the two-parameter dynamics diagrams. However, divergence between the results of these two impact cases becomes obvious in the two-parameter transition diagrams under high energy dissipation rates (i.e., small r).

References

[1] Afebu, K. O., Liu, Y., Papatheou, E. (2022) Machine learning-based rock characterisation models for rotary-percussive drilling. Nonlinear Dynamics, 109(4), 2525-2545.

[2] Afebu, K. O., Tian, J., Liu, Y., Papatheou, E., Prasad, S. (2023). AI-Assisted Dynamic Tissue Evaluation for Early Bowel Cancer Diagnosis Using a Vibrational Capsule. IEEE Robotics and Automation Letters, 8(4), 2341-2348.

[3] Okolewski, A., Blazejczyk-Okolewska, B. (2021) Hard versus soft impact modeling of vibro-impact systems with a moving base. Nonlinear Dynamics, 105, 1389-1403.

[4] Okolewski, A., Blazejczyk-Okolewska, B. (2021) Hard vs soft impacts in oscillatory systems' modeling revisited. Chaos: An Interdisciplinary Journal of Nonlinear Science, 31(8).

[5] Fan, Q., Yan, X. (2015) Self-adaptive differential evolution algorithm with discrete mutation control parameters. Expert Systems with Applications, 42(3), 1551-1572.

[6] Arora, J. S. (2004) Introduction to optimum design. Elsevier.