EJERCICIO Nº 1

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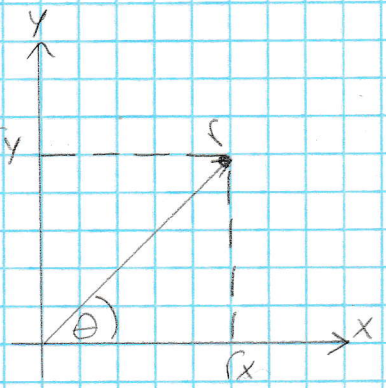
① Relacionar

Teoría: Coordenada Polar (r, θ)

$r \rightarrow$ Magnitud
$\theta \rightarrow$ Angulo



$r_y = r \operatorname{Sen}\theta \Rightarrow y = r \operatorname{Sen}\theta$

$r_x = r \operatorname{Cos}\theta \Rightarrow x = r \operatorname{Cos}\theta$

$r^2 = x^2 + y^2 \Rightarrow r = \sqrt{(r \operatorname{Sen}\theta)^2 + (r \operatorname{Cos}\theta)^2}$

$\operatorname{Tan}\theta = \frac{r_y}{r_x} \Rightarrow \operatorname{Tan}\theta = \frac{y}{x} \Rightarrow \theta = \arctan\left(\frac{y}{x}\right)$

a) $y = -4$

b) $xy = 1$

c) $x^2 + y^2 - 6x = 0$

d) $x^2 + y^2 - 4x + 2y$

e) $y^2 = 1 - 4x$

I) $r = 4\operatorname{Cos}\theta - 2\operatorname{Sen}\theta$

II) $r^2 \operatorname{Sen}^2\theta = 1 - 4r\operatorname{Cos}\theta$

III) $r = 6\operatorname{Cos}\theta$

IV) $r^2 \operatorname{Sen}(2\theta) = 2$

V) $r = -\frac{4}{\operatorname{Sen}\theta}$

$y = -4 \Rightarrow r\operatorname{Sen}\theta = -4 \Rightarrow r = -\frac{4}{\operatorname{Sen}\theta}$ a) → V)

$xy = 1 \Rightarrow r\operatorname{Cos}\theta\, r\operatorname{Sen}\theta = 1 \Rightarrow r^2 \operatorname{Sen}\theta\operatorname{Cos}\theta = 1 \Rightarrow \frac{r^2 \operatorname{Sen}(2\theta)}{2} = 1$

$\Rightarrow r^2 \operatorname{Sen}(2\theta) = 2$ b) → IV)

$x^2 + y^2 - 6x = 0 \Rightarrow (x^2 + y^2) = 6x \Rightarrow r^2 = 6r\operatorname{Cos}\theta \Rightarrow r = 6\operatorname{Cos}\theta$

c) → III)

$x^2 + y^2 - 4x + 2y \Rightarrow (x^2 + y^2) - 4x + 2y \Rightarrow r^2 - 4r\operatorname{Cos}\theta + 2r\operatorname{Sen}\theta$

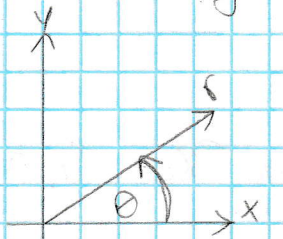
$\Rightarrow r^2 - r(4\operatorname{Cos}\theta - 2\operatorname{Sen}\theta) = 0 \Rightarrow r = 4\operatorname{Cos}\theta - 2\operatorname{Sen}\theta$ d) → I)

② Hallar la Distancia entre dos puntos

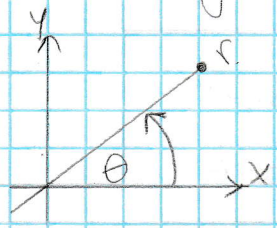
$P_1(-3, 75°)$ $P_2(5, 45°)$

Teoría: Puntos (r, θ) Si $r(+)$ y $\theta(+)$ se grafica así:

"Normal"

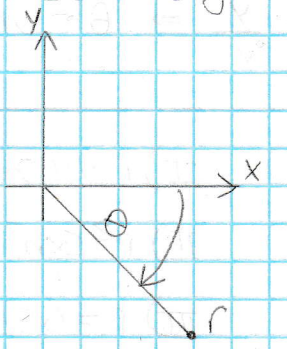


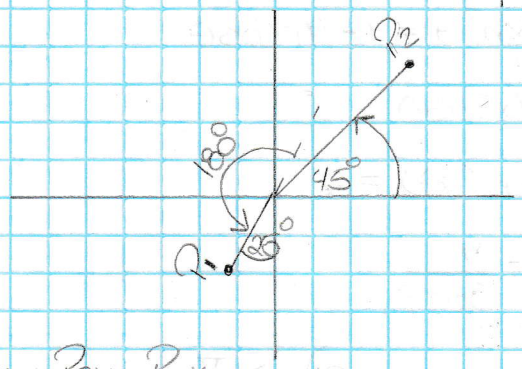
Si $r(-)$ y $\theta(+)$ se grafica así:



OJO: Se suma 180° a la ubicación del punto

Si $r(+)$ y $\theta(-)$ se grafica así:





$P_{2x} = r\cos\theta = 5\sqrt{2}/2$

$P_{2y} = r\sin\theta = 5\sqrt{2}/2$

$P_{1x} = r\sin\theta = -1.2678$

$P_{1y} = r\cos\theta = -2.7189$

$R_x = P_{2x} - P_{1x}$

$R_y = P_{2y} - P_{1y}$

$R_x = 4.80$

$R_y = 5.715$

$R = \sqrt{R_x^2 + R_y^2} \Rightarrow R = 7.46 \approx 7$

③ Numero de Ptos de Intersección

$r = 4(1+\operatorname{Sen}\theta)$; $r(1-\operatorname{Sen}\theta) = 3$

$$r = \frac{3}{(1-\operatorname{Sen}\theta)}$$

1º Paso → Igualar Ambas curvas

$$4(1+\operatorname{Sen}\theta) = \frac{3}{(1-\operatorname{Sen}\theta)}$$

2º Paso → Aplicar Identidades Trigonométricas

$4(1-\operatorname{Sen}^2\theta) = 3 \Rightarrow \operatorname{Cos}^2\theta = \frac{3}{4} \Rightarrow \operatorname{Cos}\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \operatorname{arcCos}\left(\frac{\sqrt{3}}{2}\right)$

$\Rightarrow \theta = \frac{\pi}{6}$ v $\theta = \frac{11\pi}{6}$

3º Reemplazar en Y de las Curves

$r = 4\left(1+\frac{1}{2}\right) \Rightarrow r = 4\left(\frac{3}{2}\right) \Rightarrow r = 6$

$r = 4\left(1-\frac{1}{2}\right) \Rightarrow r = 4\left(\frac{1}{2}\right) \Rightarrow r = 2$

4º Hallar los Puntos (r, θ)

$P_1 \left(6, \pi/6\right)$ $P_3 \left(6, 11\pi/6\right)$
$P_2 \left(2, \pi/6\right)$ $P_4 \left(2, 11\pi/6\right)$

Total 4 Puntos