DEFINICIÓN DE LA DERIVADA

Formas de expresar una derivada: $y' = \frac{dy}{dx} = f'(x) = \frac{d}{dx}\big(f(x)\big) = D_x\big(f(x)\big)$

Derivada General	Derivada en un punto
$\boldsymbol{f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}}$	$f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

- $D_x(k) = 0 \, , k \epsilon \mathbb{R}$

$$\equiv \lim_{h\to 0}\frac{k-k}{h} = \lim_{h\to 0}\frac{0}{h} = 0$$

- $D_x(x^n) = nx^{n-1} \, , n \epsilon \mathbb{R}$

Derivada general $f'(x)$

$$\equiv \lim_{h\to 0}\frac{(x+h)^n-(x)^n}{h} = \lim_{h\to 0}\frac{[(x+h)-(x)][(x+h)^{n-1}+(x+h)^{n-2}(x)+(x+h)^{n-3}(x)^2+(x+h)^{n-4}(x)^3+\cdots+(x)^{n-1}]}{h}$$

$$\equiv \lim_{h\to 0}\frac{[h][(x+h)^{n-1}+(x+h)^{n-2}(x)+(x+h)^{n-3}(x)^2+(x+h)^{n-4}(x)^3+\cdots+(x)^{n-1}]}{h}$$

$$\equiv \lim_{h\to 0}\frac{[\cancel{h}][(x+h)^{n-1}+(x+h)^{n-2}(x)+(x+h)^{n-3}(x)^2+(x+h)^{n-4}(x)^3+\cdots+(x)^{n-1}]}{\cancel{h}}$$

$$\equiv \lim_{h\to 0}[(x+h)^{n-1}+(x+h)^{n-2}(x)+(x+h)^{n-3}(x)^2+(x+h)^{n-4}(x)^3+\cdots+(x)^{n-1}]$$

$$\equiv \left[\big(x+(0)\big)^{n-1}+\big(x+(0)\big)^{n-2}(x)+\big(x+(0)\big)^{n-3}(x)^2+\big(x+(0)\big)^{n-4}(x)^3+\cdots+(x)^{n-1}\right]$$

$$\equiv [(x)^{n-1}+(x)^{n-2}(x)+(x)^{n-3}(x)^2+(x)^{n-4}(x)^3+\cdots+(x)^{n-1}]$$

$$\equiv \left[\underbrace{(x)^{n-1}+(x)^{n-1}+(x)^{n-1}+(x)^{n-1}+\cdots+(x)^{n-1}}_{n}\right] = nx^{n-1}$$

Derivada en el punto $f'(a)$

$$\equiv \lim_{x\to a}\frac{(x)^n-(a)^n}{x-a} = \lim_{x\to a}\frac{[x-a][(x)^{n-1}+(x)^{n-2}(a)+(x)^{n-3}(a)^2+(x)^{n-4}(a)^3+\cdots+(a)^{n-1}]}{x-a}$$

$$\equiv \lim_{x\to a}\frac{[\cancel{x-a}][(x)^{n-1}+(x)^{n-2}(a)+(x)^{n-3}(a)^2+(x)^{n-4}(a)^3+\cdots+(a)^{n-1}]}{\cancel{x-a}}$$

$$\equiv \lim_{x\to a}[(x)^{n-1}+(x)^{n-2}(a)+(x)^{n-3}(a)^2+(x)^{n-4}(a)^3+\cdots+(a)^{n-1}]$$

$$\equiv [(a)^{n-1}+(a)^{n-2}(a)+(a)^{n-3}(a)^2+(a)^{n-4}(a)^3+\cdots+(a)^{n-1}]$$

$$\equiv [(a)^{n-1}+(a)^{n-1}+(a)^{n-1}+(a)^{n-1}+\cdots+(a)^{n-1}] = n(a)^{n-1}$$

- $D_x(a^x) = a^x\,ln(a)\,,\ a \in \mathbb{R}$

Derivada general $f'(x)$

$$\equiv \lim_{h\to 0}\frac{a^{x+h}-a^x}{h} = \lim_{h\to 0}\frac{a^x a^h - a^x}{h} = \lim_{h\to 0}\frac{a^x(a^h-1)}{h}$$

$$\equiv a^x\left(\lim_{h\to 0}\frac{a^h-1}{h}\right) = a^x\left(\cancel{\lim_{h\to 0}\frac{a^h-1}{h}}\right) = a^x\,ln(a)$$

Derivada en el punto $f'(c)$

Utilizando el cambio de variable de:

$$u = x - c$$
$$x = u + c$$
$$u \longrightarrow 0$$

$$\equiv \lim_{x\to c}\frac{a^x-a^c}{x-c} = \lim_{u\to 0}\frac{a^{u+c}-a^c}{u} = \lim_{u\to 0}\frac{a^u a^c - a^c}{u} = \lim_{u\to 0}\frac{a^c(a^u-1)}{u}$$

$$\equiv a^c\left(\lim_{h\to 0}\frac{a^u-1}{u}\right) = a^c\left(\cancel{\lim_{h\to 0}\frac{a^u-1}{u}}\right) = a^c\,ln(a)$$

- $D_x(ln(x)) = \frac{1}{x}$

Derivada general $f'(x)$

$$\equiv \lim_{h\to 0}\frac{ln(x+h)-ln(x)}{h} = \lim_{h\to 0}\frac{1}{h}\left[ln\left(\frac{x+h}{x}\right)\right] = \lim_{h\to 0}\frac{1}{h}\left[ln\left(1+\frac{h}{x}\right)\right] = \lim_{h\to 0}\left[ln\left(1+\frac{h}{x}\right)^{\frac{1}{h}}\right]$$

$$\equiv ln\left[\lim_{h\to 0}\left(1+\frac{h}{x}\right)^{\frac{1}{h}}\right] = ln\left[\left(\lim_{h\to 0}\left(1+\frac{h}{x}\right)^{\frac{1}{\frac{h}{x}}}\right)^{\frac{1}{x}}\right] = ln\left[\left(\underbrace{\lim_{h\to 0}\left(1+\frac{h}{x}\right)^{\frac{1}{\frac{h}{x}}}}_{e}\right)^{\frac{1}{x}}\right] = ln\left[e^{\frac{1}{x}}\right] = \frac{1}{x}$$

Derivada en el punto $f'(a)$

Utilizando el cambio de variable de:

$$u = x - a$$
$$x = u + a$$
$$u \longrightarrow 0$$

$$\equiv \lim_{x\to a}\frac{ln(x)-ln(a)}{x-a} = \lim_{x\to a}\frac{1}{x-a}\left[ln\left(\frac{x}{a}\right)\right] = \lim_{u\to 0}\frac{1}{u}\left[ln\left(\frac{u+a}{a}\right)\right]$$

$$\equiv \lim_{u\to 0}\left[ln\left(1+\frac{u}{a}\right)^{\frac{1}{u}}\right] = ln\left[\lim_{h\to 0}\left(1+\frac{u}{a}\right)^{\frac{1}{u}}\right] = ln\left[\left(\lim_{h\to 0}\left(1+\frac{u}{a}\right)^{\frac{1}{\frac{u}{a}}}\right)^{\frac{1}{a}}\right]$$

$$\equiv ln\left[\left(\underbrace{\lim_{h\to 0}\left(1+\frac{u}{a}\right)^{\frac{1}{\frac{u}{a}}}}_{e}\right)^{\frac{1}{a}}\right] = ln\left[e^{\frac{1}{a}}\right] = \frac{1}{a}$$

➢ $D_x(log_a(x)) = \frac{1}{x\,ln(a)},\ a \epsilon \mathbb{R}$

Derivada general $f'(x)$

$$\equiv \lim_{h\to 0}\frac{log_a(x+h)-log_a(x)}{h} = \lim_{h\to 0}\frac{1}{h}\left[log_a\left(\frac{x+h}{x}\right)\right] = \lim_{h\to 0}\frac{1}{h}\left[\frac{ln\left(\frac{x+h}{x}\right)}{ln(a)}\right] = \frac{1}{ln(a)}\lim_{h\to 0}\frac{1}{h}\left[ln\left(\frac{x+h}{x}\right)\right]$$

$$\equiv \frac{1}{ln(a)}\ \lim_{h\to 0}\frac{1}{h}\left[ln\left(1+\frac{h}{x}\right)\right] = \frac{1}{ln(a)}\ \lim_{h\to 0}\left[ln\left(1+\frac{h}{x}\right)^{\frac{1}{h}}\right] = \frac{1}{ln(a)}\ ln\left[\lim_{h\to 0}\left(1+\frac{h}{x}\right)^{\frac{1}{h}}\right]$$

$$\equiv \frac{1}{ln(a)}\,ln\left[\left(\lim_{h\to 0}\left(1+\frac{h}{x}\right)^{\frac{1}{\frac{h}{x}}}\right)^{\frac{1}{x}}\right] = \frac{1}{ln(a)}\,ln\left[\left(\underbrace{\lim_{h\to 0}\left(1+\frac{h}{x}\right)^{\frac{1}{\frac{h}{x}}}}_{e}\right)^{\frac{1}{x}}\right] = \frac{1}{ln(a)}\,ln\left[\mathrm{e}^{\frac{1}{x}}\right] = \frac{1}{x\,ln(a)}$$

Derivada en el punto $f'(c)$

$$\equiv \lim_{x\to c}\frac{log_a(x)-log_a(c)}{x-c} = \lim_{x\to c}\frac{1}{x-c}\left[log_a\left(\frac{x}{c}\right)\right] = \lim_{u\to 0}\frac{1}{u}\left[log_a\left(\frac{u+c}{c}\right)\right] = \lim_{u\to 0}\frac{1}{u}\left[\frac{ln\left(\frac{u+c}{c}\right)}{ln(a)}\right]$$

Utilizando el cambio de variable de:

$u = x - c$

$x = u + c$

$u \longrightarrow 0$

$$\equiv \frac{1}{ln(a)}\lim_{u\to 0}\frac{1}{u}\left[ln\left(\frac{u+c}{c}\right)\right] = \frac{1}{ln(a)}\ \lim_{u\to 0}\frac{1}{u}\left[ln\left(1+\frac{u}{c}\right)\right]$$

$$\equiv \frac{1}{ln(a)}\ \lim_{h\to 0}\left[ln\left(1+\frac{u}{c}\right)^{\frac{1}{c}}\right] = \frac{1}{ln(a)}\ ln\left[\lim_{h\to 0}\left(1+\frac{u}{c}\right)^{\frac{1}{c}}\right]$$

$$\equiv \frac{1}{ln(a)}\,ln\left[\left(\lim_{h\to 0}\left(1+\frac{u}{c}\right)^{\frac{1}{\frac{u}{c}}}\right)^{\frac{1}{c}}\right] = \frac{1}{ln(a)}\,ln\left[\left(\underbrace{\lim_{h\to 0}\left(1+\frac{u}{c}\right)^{\frac{1}{\frac{u}{c}}}}_{e}\right)^{\frac{1}{c}}\right] = \frac{1}{ln(a)}\,ln\left[\mathrm{e}^{\frac{1}{c}}\right] = \frac{1}{c\,ln(a)}$$

➢ $D_x(\mathrm{sen}(x)) = \cos(x)$

Derivada general $f'(x)$

$$\equiv \lim_{h\to 0}\frac{sen(x+h)-\mathrm{sen}(x)}{h} = \lim_{h\to 0}\frac{[sen(x)\cos(h)+sen(h)\cos(x)]-\mathrm{sen}(x)}{h}$$

$$\equiv \lim_{h\to 0}\frac{sen(x)\cos(h)+sen(h)\cos(x)-\mathrm{sen}(x)}{h} = \lim_{h\to 0}\frac{sen(x)(\cos(h)-1)+sen(h)\cos(x)}{h}$$

$$\equiv \lim_{h\to 0}\frac{-sen(x)(1-cos(h))+sen(h)\,cos(x)}{h} = -sen(x)\lim_{h\to 0}\frac{(1-cos(h))}{h} + cos(x)\lim_{h\to 0}\frac{sen(h)}{h}$$

$$\equiv -sen(x)\left(\cancel{\lim_{h\to 0}\frac{1-cos(h)}{h}}\right) + cos(x)\left(\cancel{\lim_{h\to 0}\frac{sen(h)}{h}}\right) = cos(x)$$

Derivada en el punto $f'(a)$

$$\equiv \lim_{x\to a}\frac{sen(x)-\mathrm{sen}(a)}{x-a} = \lim_{u\to 0}\frac{sen(u+a)-\mathrm{sen}(a)}{u} = \lim_{u\to 0}\frac{[sen(u)\,cos(a)+sen(a)\,cos(u)]-sen(a)}{u}$$

Utilizando el cambio de variable de:

$u = x - a$

$x = u + a$

$u \longrightarrow 0$

$$\equiv \lim_{u\to 0}\frac{sen(u)\,cos(a)+sen(a)\,cos(u)-\mathrm{sen}(a)}{h} = \lim_{u\to 0}\frac{sen(a)(\cos(u)-1)+sen(u)\cos(a)}{u}$$

$$\equiv \lim_{u\to 0}\frac{-sen(a)(1-cos(u))+sen(u)\,cos(a)}{u} = -sen(a)\lim_{u\to 0}\frac{(1-cos(u))}{u} + cos(a)\lim_{u\to 0}\frac{sen(u)}{u}$$

$$\equiv -sen(a)\left(\cancel{\lim_{u\to 0}\frac{1-cos(u)}{u}}\right) + cos(a)\left(\cancel{\lim_{u\to 0}\frac{sen(u)}{u}}\right) = cos(a)$$

- $D_x(\cos(x)) = -\operatorname{sen}(x)$

Derivada general $f'(x)$

$$\equiv \lim_{h\to 0}\frac{cos(x+h)-\cos(x)}{h} = \lim_{h\to 0}\frac{[cos(x)\cos(h)-sen(h)\operatorname{sen}(x)]-\cos(x)}{h}$$

$$\equiv \lim_{h\to 0}\frac{cos(x)\,cos(h)-sen(h)\,sen(x)-\cos(x)}{h} = \lim_{h\to 0}\frac{cos(x)(\cos(h)-1)-sen(h)\operatorname{sen}(x)}{h}$$

$$\equiv \lim_{h\to 0}\frac{-cos(x)(1-cos(h))-sen(h)\,sen(x)}{h} = -cos(x)\lim_{h\to 0}\frac{(1-cos(h))}{h} - sen(x)\lim_{h\to 0}\frac{sen(h)}{h}$$

$$\equiv -cos(x)\left(\lim_{h\to 0}\frac{1-cos(h)}{h}\right) - sen(x)\left(\lim_{h\to 0}\frac{sen(h)}{h}\right) = -sen(x)$$

Derivada en el punto $f'(a)$

$$\equiv \lim_{x\to a}\frac{cos(x)-cos(a)}{x-a} = \lim_{u\to 0}\frac{\cos(u+a)-\cos(a)}{u} = \lim_{u\to 0}\frac{[cos(u)\,cos(a)-sen(a)\,sen(u)]-\cos(a)}{u}$$

Utilizando el cambio de variable de:

$u = x - a$

$x = u + a$

$u \longrightarrow 0$

$$\equiv \lim_{u\to 0}\frac{cos(u)\,cos(a)-sen(a)\,sen(u)-\cos(a)}{h} = \lim_{u\to 0}\frac{cos(a)(\cos(u)-1)-sen(u)\operatorname{sen}(a)}{u}$$

$$\equiv \lim_{u\to 0}\frac{-cos(a)(1-cos(u))-sen(u)\,sen(a)}{u} = -cos(a)\lim_{u\to 0}\frac{(1-cos(u))}{u} - sens(a)\lim_{u\to 0}\frac{sen(u)}{u}$$

$$\equiv -cos(a)\left(\lim_{u\to 0}\frac{1-cos(u)}{u}\right) - sens(a)\left(\lim_{u\to 0}\frac{sen(u)}{u}\right) = -sen(a)$$

- $D_x(tan(x)) = sec^2(x)$

Derivada general $f'(x)$

$$\equiv \lim_{h\to 0}\frac{tan(x+h)-tan(x)}{h} = \lim_{h\to 0}\frac{\frac{sen(x+h)}{cos(x+h)}-\frac{sen(x)}{cos(x)}}{h} = \lim_{h\to 0}\frac{\frac{cos(x)\,sen(x+h)-sen(x)\,cos(x+h)}{cos(x+h)\,cos(x)}}{h}$$

$$\equiv \lim_{h\to 0}\frac{cos(x)[sen(x)\,cos(h)+sen(h)\,cos(x)]-sen(x)[cos(x)\,cos(h)-sen(h)\,sen(x)]}{h\,cos(x+h)\,cos(x)}$$

$$\equiv \lim_{h\to 0}\frac{cos(x)\,sen(x)\,cos(h)+sen(h)\,cos^2(x)-sen(x)\,cos(x)\,cos(h)+sen(h)\,sen^2(x)}{h\,cos(x+h)\,cos(x)}$$

$$\equiv \lim_{h\to 0}\frac{sen(h)\,cos^2(x)+sen(h)\,sen^2(x)}{h\,cos(x+h)\,cos(x)} = \lim_{h\to 0}\frac{sen(h)[cos^2(x)+sen^2(x)]}{h\,cos(x+h)\,cos(x)} = \lim_{h\to 0}\frac{sen(h)}{h\,cos(x+h)\,cos(x)}$$

$$\equiv \left[\lim_{h\to 0}\frac{sen(h)}{h}\right]\left[\lim_{h\to 0}\frac{1}{cos(x+h)\,cos(x)}\right] = \left[\lim_{h\to 0}\frac{sen(h)}{h}\right]\left[\frac{1}{cos^2(x)}\right] = \frac{1}{cos^2(x)} = sec^2(x)$$

Derivada en el punto $f'(a)$

$$\equiv \lim_{x\to a}\frac{tan(x)-tan(a)}{x-a} = \lim_{u\to 0}\frac{tan(u+a)-tan(a)}{u} = \lim_{u\to 0}\frac{\frac{sen(u+a)}{cos(u+a)}-\frac{sen(a)}{cos(a)}}{h} = \lim_{u\to 0}\frac{\frac{cos(a)\,sen(u+a)-sen(a)\,cos(u+a)}{cos(u+a)\,cos(a)}}{u}$$

Utilizando el cambio de variable de:

$u = x - a$

$x = u + a$

$u \longrightarrow 0$

$$\equiv \lim_{u\to 0}\frac{cos(a)[sen(u)\,cos(a)+sen(a)\,cos(u)]-sen(a)[cos(u)\,cos(a)-sen(a)\,sen(u)]}{u\,cos(u+a)\,cos(a)}$$

$$\equiv \lim_{u\to 0}\frac{sen(u)\,cos^2(a)+cos(a)\,sen(a)\,cos(u)-sen(a)\,cos(u)\,cos(a)+sen(u)\,sen^2(a)}{u\,cos(u+a)\,cos(a)}$$

$$\equiv \lim_{u\to 0}\frac{sen(u)\,cos^2(a)+sen(u)\,sen^2(a)}{u\,cos(u+a)\,cos(a)} = \lim_{u\to 0}\frac{sen(u)[cos^2(a)+sen^2(a)]}{u\,cos(u+a)\,cos(a)} = \lim_{u\to 0}\frac{sen(u)}{u\,cos(u+a)\,cos(a)}$$

$$\equiv \left[\lim_{u\to 0}\frac{sen(u)}{u}\right]\left[\lim_{u\to 0}\frac{1}{cos(u+a)\,cos(a)}\right] = \left[\lim_{u\to 0}\frac{sen(u)}{u}\right]\left[\frac{1}{cos^2(a)}\right] = \frac{1}{cos^2(a)} = sec^2(a)$$

➢ $D_x(cot(x)) = -\,csc^2(x)$

Derivada general $f'(x)$

$$\equiv \lim_{h\to 0}\frac{cot(x+h)-cot(x)}{h} = \lim_{h\to 0}\frac{\frac{cos(x+h)}{sen(x+h)}-\frac{cos(x)}{sen(x)}}{h} = \lim_{h\to 0}\frac{\frac{sen(x)\,cos(x+h)-cos(x)\,sen(x+h)}{sen(x+h)\,sen(x)}}{h}$$

$$\equiv \lim_{h\to 0}\frac{sen(x)[cos(x)\,cos(h)-sen(h)\,sen(x)]-cos(x)[sen(x)\,cos(h)+sen(h)\,cos(x)]}{h\,sen(x+h)\,sen(x)}$$

$$\equiv \lim_{h\to 0}\frac{sen(x)\,cos(x)\,cos(h)-sen(h)\,sen^2(x)-cos(x)\,sen(x)\,cos(h)-sen(h)\,cos^2(x)}{h\,sen(x+h)\,sen(x)}$$

$$\equiv \lim_{h\to 0}\frac{-\,sen(h)\,sen^2(x)-sen(h)\,cos^2(x)}{h\,sen(x+h)\,sen(x)} = \lim_{h\to 0}\frac{-\,sen(h)[sen^2(x)+cos^2(x)]}{h\,sen(x+h)\,sen(x)} = \lim_{h\to 0}\frac{-\,sen(h)}{h\,sen(x+h)\,sen(x)}$$

$$\equiv \left[-\lim_{h\to 0}\frac{sen(h)}{h}\right]\left[\lim_{h\to 0}\frac{1}{sen(x+h)\,sen(x)}\right] = -\left[\lim_{h\to 0}\frac{sen(h)}{h}\right]\left[\frac{1}{sen^2(x)}\right] = -\frac{1}{sen^2(x)} = -\,csc^2(x)$$

Derivada en el punto $f'(a)$

$$\equiv \lim_{x\to a}\frac{cot(x)-\cot(a)}{x-a} = \lim_{u\to 0}\frac{cot(u+a)-cot(a)}{u} = \lim_{u\to 0}\frac{\frac{cos(u+a)}{sen(u+a)}-\frac{cos(a)}{sen(a)}}{h} = \lim_{u\to 0}\frac{\frac{sen(a)\,cos(u+a)-\cos(a)\,\mathrm{sen}(u+a)}{sen(u+a)\,sen(a)}}{u}$$

Utilizando el cambio de variable de:

$u = x - a$

$x = u + a$

$u \longrightarrow 0$

$$\equiv \lim_{u\to 0}\frac{sen(a)[cos(u)\,cos(a)-sen(a)\,sen(u)]-cos(a)[sen(u)\,cos(a)+sen(a)\,cos(u)]}{u\,sen(u+a)\,sen(a)}$$

$$\equiv \lim_{u\to 0}\frac{sen(a)\,cos(a)\,cos(u)-sen(u)\,sen^2(a)-sen(u)\,cos^2(a)-cos(a)\,sen(a)\,cos(u)}{u\,sen(u+a)\,sen(a)}$$

$$\equiv \lim_{u\to 0}\frac{-\,sen(u)\,cos^2(a)-sen(u)\,sen^2(a)}{u\,sen(u+a)\,sen(a)} = -\lim_{u\to 0}\frac{sen(u)[cos^2(a)+sen^2(a)]}{u\,sen(u+a)\,sen(a)} = -\lim_{u\to 0}\frac{sen(u)}{u\,sen(u+a)\,sen(a)}$$

$$\equiv -\left[\lim_{u\to 0}\frac{sen(u)}{u}\right]\left[\lim_{u\to 0}\frac{1}{sen(u+a)\,sen(a)}\right] = -\left[\lim_{h\to 0}\frac{sen(u)}{u}\right]\left[\frac{1}{sen^2(a)}\right] = -\frac{1}{sen^2(a)} = -\,csc^2(a)$$

➢ $D_x(sec(x)) = sec(x)\,tan(x)$

Derivada general $f'(x)$

$$\equiv \lim_{h\to 0}\frac{sec(x+h)-sec(x)}{h} = \lim_{h\to 0}\frac{\frac{1}{cos(x+h)}-\frac{1}{cos(x)}}{h} = \lim_{h\to 0}\frac{\frac{cos(x)-cos(x+h)}{cos(x+h)\,cos(x)}}{h}$$

$$\equiv \lim_{h\to 0}\frac{cos(x)-[cos(x)\,cos(h)-sen(h)\,sen(x)]}{h\,cos(x+h)\,cos(x)} = \lim_{h\to 0}\frac{cos(x)-cos(x)\,cos(h)+sen(h)\,sen(x)}{h\,cos(x+h)\,cos(x)}$$

$$\equiv \lim_{h\to 0}\frac{cos(x)(1-cos(h))+sen(h)\,sen(x)}{h\,cos(x+h)\,cos(x)} = \lim_{h\to 0}\left[\frac{cos(x)(1-cos(h))}{h\,cos(x+h)\,cos(x)}+\frac{sen(h)\,sen(x)}{h\,cos(x+h)\,cos(x)}\right]$$

$$\equiv \left[\left(\lim_{h\to 0}\frac{1-cos(h)}{h}\right)\left(\lim_{h\to 0}\frac{1}{cos(x+h)}\right)\right]+\left[\left(\lim_{h\to 0}\frac{sen(h)}{h}\right)\left(\lim_{h\to 0}\frac{sen(x)}{cos(x+h)\,cos(x)}\right)\right]$$

$$\equiv \left[\left(\lim_{h\to 0}\frac{1-cos(h)}{h}\right)\left(\frac{1}{cos(x)}\right)\right]+\left[\left(\lim_{h\to 0}\frac{sen(h)}{h}\right)\left(\frac{sen(x)}{cos(x)\,cos(x)}\right)\right] = \frac{sen(x)}{cos(x)\,cos(x)} = sec(x)\,tan(x)$$

Derivada en el punto $f'(a)$

$$\equiv \lim_{x\to a}\frac{sec(x)-sec(a)}{x-a} = \lim_{u\to 0}\frac{sec(u+a)-sec(a)}{u} = \lim_{u\to 0}\frac{\frac{1}{cos(u+a)}-\frac{1}{cos(a)}}{u} = \lim_{u\to 0}\frac{\frac{cos(a)-cos(u+a)}{cos(u+a)\,cos(a)}}{u}$$

Utilizando el cambio de variable de:

$u = x - a$

$x = u + a$

$u \longrightarrow 0$

$$\equiv \lim_{u\to 0}\frac{cos(a)-[cos(u)\,cos(a)-sen(a)\,sen(u)]}{u\,cos(u+a)\,cos(a)} = \lim_{u\to 0}\frac{cos(a)-cos(u)\,cos(a)+sen(a)\,sen(u)}{u\,cos(u+a)\,cos(a)}$$

$$\equiv \lim_{u\to 0}\frac{cos(a)(1-cos(u))+sen(a)\,sen(u)}{u\,cos(u+a)\,cos(a)} = \lim_{u\to 0}\left[\frac{cos(a)(1-cos(u))}{u\,cos(u+a)\,cos(a)}+\frac{sen(a)\,sen(u)}{u\,cos(u+a)\,cos(a)}\right]$$

$$\equiv \left[\left(\lim_{u\to 0}\frac{1-cos(u)}{u}\right)\left(\lim_{u\to 0}\frac{1}{cos(u+a)}\right)\right]+\left[\left(\lim_{u\to 0}\frac{sen(u)}{u}\right)\left(\lim_{u\to 0}\frac{sen(a)}{cos(u+a)\,cos(a)}\right)\right]$$

$$\equiv \left[\left(\lim_{u\to 0}\frac{1-cos(u)}{u}\right)\left(\frac{1}{cos(a)}\right)\right]+\left[\left(\lim_{u\to 0}\frac{sen(u)}{u}\right)\left(\frac{sen(a)}{cos(a)\,cos(a)}\right)\right] = \frac{sen(x)}{cos(x)\,cos(x)} = sec(x)\,tan(x)$$

"NO ES LO MISMO APRENDER QUE ENTENDER. TODOS LOS TRIUNFOS NACEN CUANDO NOS ATREVEMOS A EMPEZAR"

Yandry Tierra G.

➢ $D_x(csc(x)) = -csc(x)\,cot(x)$

Derivada general $\boldsymbol{f'(x)}$

$$\equiv \lim_{h\to 0}\frac{csc(x+h)-csc(x)}{h} = \lim_{h\to 0}\frac{\frac{1}{sen(x+h)}-\frac{1}{sen(x)}}{h} = \lim_{h\to 0}\frac{\frac{sen(x)-sen(x+h)}{sen(x+h)\,sen(x)}}{h}$$

$$\equiv \lim_{h\to 0}\frac{sen(x)-[sen(x)\,cos(h)+sen(h)\,cos(x)]}{h\,sen(x+h)\,sen(x)} = \lim_{h\to 0}\frac{sen(x)-sen(x)\,cos(h)-sen(h)\,cos(x)}{h\,sen(x+h)\,sen(x)}$$

$$\equiv \lim_{h\to 0}\frac{sen(x)(1-cos(h))-sen(h)\,cos(x)}{h\,sen(x+h)\,sen(x)} = \lim_{h\to 0}\left[\frac{sen(x)(1-cos(h))}{h\,sen(x+h)\,sen(x)}-\frac{sen(h)\,cos(x)}{h\,sen(x+h)\,sen(x)}\right]$$

$$\equiv \left[\left(\lim_{h\to 0}\frac{1-cos(h)}{h}\right)\left(\lim_{h\to 0}\frac{1}{sen(x+h)}\right)\right]-\left[\left(\lim_{h\to 0}\frac{sen(h)}{h}\right)\left(\lim_{h\to 0}\frac{cos(x)}{sen(x+h)\,sen(x)}\right)\right]$$

$$\equiv \left[\left(\lim_{h\to 0}\frac{1-cos(h)}{h}\right)\left(\frac{1}{cos(x)}\right)\right]-\left[\left(\lim_{h\to 0}\frac{sen(h)}{h}\right)\left(\frac{cos(x)}{sen(x)\,sen(x)}\right)\right] = -\frac{cos(x)}{sen(x)\,sen(x)} = -csc(x)\,cot(x)$$

Derivada en el punto $\boldsymbol{f'(a)}$

$$\equiv \lim_{x\to a}\frac{csc(x)-csc(a)}{x-a} = \lim_{u\to 0}\frac{csc(u+a)-csc(a)}{u} = \lim_{u\to 0}\frac{\frac{1}{sen(u+a)}-\frac{1}{sen(a)}}{u} = \lim_{u\to 0}\frac{\frac{sen(a)-sen(u+a)}{sen(u+a)\,sen(a)}}{u}$$

Utilizando el cambio de variable de:
$u = x - a$
$x = u + a$
$u \longrightarrow 0$

$$\equiv \lim_{u\to 0}\frac{sen(a)-[sen(u)\,cos(a)+sen(a)\,cos(u)]}{u\,sen(u+a)\,sen(a)} = \lim_{u\to 0}\frac{sen(a)-sen(u)\,cos(a)-sen(a)\,cos(u)}{u\,sen(u+a)\,sen(a)}$$

$$\equiv \lim_{u\to 0}\frac{sen(a)(1-cos(u))-sen(u)\,cos(a)}{u\,sen(u+a)\,sen(a)} = \lim_{u\to 0}\left[\frac{sen(a)(1-cos(u))}{u\,sen(u+a)\,sen(a)}-\frac{sen(u)\,cos(a)}{u\,sen(u+a)\,sen(a)}\right]$$

$$\equiv \left[\left(\lim_{u\to 0}\frac{1-cos(u)}{u}\right)\left(\lim_{u\to 0}\frac{1}{sen(u+a)}\right)\right]-\left[\left(\lim_{u\to 0}\frac{sen(u)}{u}\right)\left(\lim_{u\to 0}\frac{cos(a)}{sen(u+a)\,sen(a)}\right)\right]$$

$$\equiv \left[\left(\lim_{u\to 0}\frac{1-cos(u)}{u}\right)\left(\frac{1}{sen(a)}\right)\right]-\left[\left(\lim_{u\to 0}\frac{sen(u)}{u}\right)\left(\frac{cos(a)}{sen(a)\,sen(a)}\right)\right] = -\frac{cos(a)}{sen(a)\,sen(a)} = -csc(x)\,cot(x)$$

➢ $D_x\big(f(x)+g(x)\big) = f'(x)+g'(x)$

Derivada general $\boldsymbol{f'(x)}$

$$\equiv \lim_{h\to 0}\frac{\big(f(x+h)+g(x+h)\big)-\big(f(x)+g(x)\big)}{h} = \left[\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \pm \lim_{h\to 0}\frac{g(x+h)-g(x)}{h}\right] = f'(x) \pm g'(x)$$

Derivada en el punto $\boldsymbol{f'(a)}$

$$\equiv \lim_{x\to a}\frac{\big(f(x)+g(x)\big)-\big(f(a)+g(a)\big)}{x-a} = \left[\lim_{x\to a}\frac{f(x)-f(a)}{x-a} \pm \lim_{x\to a}\frac{g(x)-g(a)}{x-a}\right] = f'(a) \pm g'(a)$$

➢ $D_x\big(f(x)-g(x)\big) = f'(x)-g'(x)$

Derivada general $\boldsymbol{f'(x)}$

$$\equiv \lim_{h\to 0}\frac{\big(f(x+h)-g(x+h)\big)-\big(f(x)-g(x)\big)}{h} = \left[\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} - \lim_{h\to 0}\frac{g(x+h)-g(x)}{h}\right] = f'(x) - g'(x)$$

Derivada en el punto $\boldsymbol{f'(a)}$

$$\equiv \lim_{x\to a}\frac{\big(f(x)+g(x)\big)-\big(f(a)+g(a)\big)}{x-a} = \left[\lim_{x\to a}\frac{f(x)-f(a)}{x-a} \pm \lim_{x\to a}\frac{g(x)-g(a)}{x-a}\right] = f'(a) \pm g'(a)$$

- $D_x\big(f(x)g(x)\big) = f'(x)g(x) + g'(x)f(x)$

Derivada general $f'(x)$

$$\equiv \lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x)}{h} = \lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x+h)g(x)+f(x+h)g(x)-f(x)g(x)}{h}$$

$$\equiv \left[\lim_{h\to 0}\frac{f(x+h)g(x)-f(x)g(x)}{h} + \lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x+h)g(x)}{h}\right] = \left[\lim_{h\to 0}\frac{g(x)\big(f(x+h)-f(x)\big)}{h} + \lim_{h\to 0}\frac{f(x+h)\big(g(x+h)-g(x)\big)}{h}\right]$$

$$\equiv \left[g(x)\left(\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\right) + \left(\lim_{h\to 0} f(x+h)\right)\left(\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}\right)\right] = f'(x)g(x) + g'(x)f(x)$$

Derivada en el punto $f'(a)$

$$\equiv \lim_{x\to a}\frac{f(x)g(x)-f(a)g(a)}{x-a} = \lim_{x\to a}\frac{f(x)g(x)-f(a)g(x)+f(a)g(x)-f(a)g(a)}{x-a}$$

$$\equiv \left[\lim_{x\to a}\frac{f(x)g(x)-f(a)g(x)}{x-a} + \lim_{x\to a}\frac{f(a)g(x)-f(a)g(a)}{h}\right] = \left[\lim_{x\to a}\frac{g(x)\big(f(x)-f(a)\big)}{x-a} + \lim_{x\to a}\frac{f(a)\big(g(x)-g(a)\big)}{x-a}\right]$$

$$\equiv \left[\left(\lim_{x\to a} g(a)\right)\left(\lim_{x\to a}\frac{f(x)-f(a)}{x-a}\right) + f(a)\left(\lim_{x\to a}\frac{g(x)-g(a)}{x-a}\right)\right] = f'(a)g(a) + g'(a)f(a)$$

- $D_x\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x)-f(x)g'(x)}{g^2(x)}$

Derivada general $f'(x)$

$$\equiv \lim_{h\to 0}\frac{\frac{f(x+h)}{g(x+h)}-\frac{f(x)}{g(x)}}{h} = \lim_{h\to 0}\frac{\frac{f(x+h)g(x)-f(x)g(x+h)}{g(x+h)g(x)}}{h} = \lim_{h\to 0}\frac{f(x+h)g(x)-f(x)g(x+h)}{h\,g(x+h)g(x)}$$

$$\equiv \lim_{h\to 0}\frac{f(x+h)g(x)-g(x)f(x)+g(x)f(x)-f(x)g(x+h)}{h\,g(x+h)g(x)} = \left[\lim_{h\to 0}\frac{f(x+h)g(x)-g(x)f(x)}{h\,g(x+h)g(x)} + \lim_{h\to 0}\frac{g(x)f(x)-f(x)g(x+h)}{h\,g(x+h)g(x)}\right]$$

$$\equiv \left[\lim_{h\to 0}\frac{g(x)[f(x+h)-f(x)]}{h\,g(x+h)g(x)} + \lim_{h\to 0}\frac{-f(x)[g(x+h)-g(x)]}{h\,g(x+h)g(x)}\right] = \left[\lim_{h\to 0}\frac{g(x)[f(x+h)-f(x)]}{h\,g(x+h)g(x)} - \lim_{h\to 0}\frac{f(x)[g(x+h)-g(x)]}{h\,g(x+h)g(x)}\right]$$

$$\equiv \left[\left(\lim_{h\to 0}\frac{g(x)}{g(x+h)g(x)}\right)\left(\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\right) - \left(\lim_{h\to 0}\frac{f(x)}{g(x+h)g(x)}\right)\left(\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}\right)\right]$$

$$\equiv \left[\left(\frac{g(x)}{g^2(x)}\right)\left(\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\right) - \left(\frac{f(x)}{g^2(x)}\right)\left(\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}\right)\right]$$

$$\equiv \left(\frac{g(x)}{g^2(x)}\right)\big(f'(x)\big) - \left(\frac{f(x)}{g^2(x)}\right)\big(g'(x)\big) = \frac{f'(x)g(x)}{g^2(x)} - \frac{f(x)g'(x)}{g^2(x)} = \frac{f'(x)g(x)-f(x)g'(x)}{g^2(x)}$$

Derivada en el punto $f'(a)$

$$\equiv \lim_{x\to a}\frac{\frac{f(x)}{g(x)}-\frac{f(a)}{g(a)}}{x-a} = \lim_{x\to a}\frac{\frac{f(x)g(a)-f(a)g(x)}{g(x)g(a)}}{x-a} = \lim_{x\to a}\frac{f(x)g(a)-f(a)g(x)}{(x-a)\,g(x)g(a)}$$

$$\equiv \lim_{x\to a}\frac{f(x)g(a)-g(x)f(x)+g(x)f(x)-f(a)g(x)}{(x-a)\,g(x)g(a)} = \left[\lim_{x\to a}\frac{f(x)g(x)-g(x)f(a)}{(x-a)\,g(x)g(a)} + \lim_{x\to a}\frac{g(a)f(x)-f(x)g(x)}{(x-a)\,g(x)g(a)}\right]$$

$$\equiv \left[\lim_{x\to a}\frac{g(x)[f(x)-f(a)]}{(x-a)\,g(x)g(a)} + \lim_{x\to a}\frac{-f(x)[g(x)-g(a)]}{(x-a)\,g(x)g(a)}\right] = \left[\lim_{x\to a}\frac{g(x)[f(x)-f(a)]}{(x-a)\,g(x)g(a)} - \lim_{x\to a}\frac{f(x)[g(x)-g(a)]}{(x-a)\,g(x)g(a)}\right]$$

$$\equiv \left[\left(\lim_{x\to a}\frac{g(x)}{g(x)g(a)}\right)\left(\lim_{x\to a}\frac{f(x)-f(a)}{x-a}\right) - \left(\lim_{h\to 0}\frac{f(x)}{g(x)g(a)}\right)\left(\lim_{x\to a}\frac{g(x)-g(a)}{x-a}\right)\right]$$

$$\equiv \left[\left(\frac{g(a)}{g^2(a)}\right)\left(\lim_{x\to a}\frac{f(x)-f(a)}{x-a}\right) - \left(\frac{f(a)}{g^2(a)}\right)\left(\lim_{x\to a}\frac{g(x)-g(a)}{x-a}\right)\right]$$

$$\equiv \left(\frac{g(a)}{g^2(a)}\right)\big(f'(a)\big) - \left(\frac{f(a)}{g^2(a)}\right)\big(g'(a)\big) = \frac{f'(a)g(a)}{g^2(a)} - \frac{f(a)g'(a)}{g^2(a)} = \frac{f'(a)g(a)-f(a)g'(a)}{g^2(a)}$$

- $D_x\big(arcsen(x)\big) = \frac{1}{\sqrt{1-x^2}}$

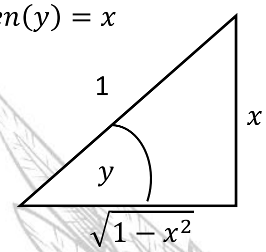
$$y = arcsen(x)$$
$$sen(y) = x$$

Derivando la expresión:

$$cos(y)\,y' = 1$$
$$y' = \frac{1}{cos(y)}$$

Sabiendo que:

$sen(y) = x$



$$y' = \frac{1}{cos(y)}$$
$$y' = \frac{1}{\sqrt{1-x^2}}$$

- $D_x\big(arccos(x)\big) = -\frac{1}{\sqrt{1-x^2}}$

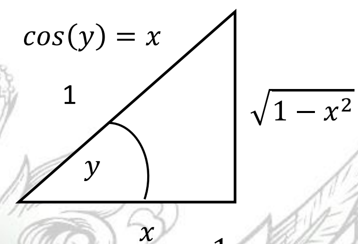
$$y = arccos(x)$$
$$cos(y) = x$$

Derivando la expresión:

$$-sen(y)\,y' = 1$$
$$y' = -\frac{1}{sen(y)}$$

Sabiendo que:

$cos(y) = x$



$$y' = -\frac{1}{sen(y)}$$
$$y' = -\frac{1}{\sqrt{1-x^2}}$$

- $D_x\big(arctan(x)\big) = \frac{1}{1+x^2}$

$$y = arctan(x)$$
$$tan(y) = x$$

Derivando la expresión:

$$sec^2(y)\,y' = 1$$
$$y' = \frac{1}{sec^2(y)}$$

Sabiendo que:

$$sec^2(y) = 1 + tan^2(y)$$
$$y' = \frac{1}{1 + tan^2(y)}$$
$$y' = \frac{1}{1 + x^2}$$

- $D_x\big(arccot(x)\big) = -\frac{1}{1+x^2}$

$$y = arccot(x)$$
$$cot(y) = x$$

Derivando la expresión:

$$-csc^2(y)\,y' = 1$$
$$y' = -\frac{1}{csc^2(y)}$$

Sabiendo que:

$$csc^2(y) = 1 + cot^2(y)$$
$$y' = -\frac{1}{1 + cot^2(y)}$$
$$y' = \frac{1}{1 + x^2}$$

- $D_x\big(arcsec(x)\big) = \frac{1}{x\sqrt{x^2-1}}$

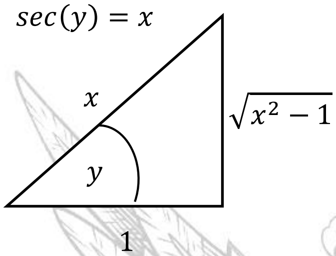
$$y = arcsec(x)$$
$$sec(y) = x$$

Derivando la expresión:

$$sec(y)tan(y)\, y' = 1$$
$$y' = \frac{1}{sec(y)tan(y)}$$

Sabiendo que:

$sec(y) = x$



$$y' = \frac{1}{sec(y)tan(y)}$$
$$y' = \frac{1}{x\sqrt{x^2-1}}$$

- $D_x\big(arccsc(x)\big) = -\frac{1}{x\sqrt{x^2-1}}$

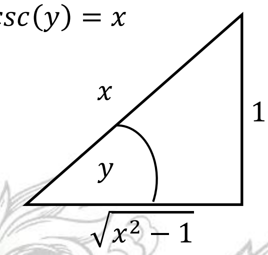
$$y = arccsc(x)$$
$$csc(y) = x$$

Derivando la expresión:

$$-csc(y)cot(y)\, y' = 1$$
$$y' = -\frac{1}{csc(y)cot(y)}$$

Sabiendo que:

$csc(y) = x$



$$y' = -\frac{1}{csc(y)cot(y)}$$
$$y' = -\frac{1}{x\sqrt{x^2-1}}$$